Status of effective translation of complicated forms in FEniCS

The UFLACS project

Martin Sandve Alnæs

Center for Biomedical Computing,
Simula Research Laboratory,
Oslo, Norway

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```
F_glob = I + grad(u)
F = variable(R.T*F_glob*R)
E = 0.5*(F.T*F - I)
J = det(F)
f=0; s=1; n=2
W = (bff*E[f,f]**2
     + bxx*(E[n,n]**2 + E[s,s]**2 + E[n,s]**2)
     + bfx*(E[f,n]**2 + E[n,f]**2 + E[f,s]**2 + E[s,f]**2))
psi = 0.5*K*(exp(W) - 1) + Ccompr*(J*ln(J) - J + 1)
```

```
s[27] = w0_c5 * s[4] + (w0_c3 * w1_d001_c0 + w0_c4 * w1_d001_c1);
s[28] = w0_c5 * w1_d100_c2 + (w0_c3 * s[6] + w0_c4 * w1_d100_c1);
s[29] = w0_c5 * w1_d010_c2 + (w0_c3 * w1_d010_c0 + w0_c4 * s[8]);
s[30] = w0_c2 * s[27] + (w0_c0 * s[28] + w0_c1 * s[29]);
s[31] = w0_c8 * s[27] + (w0_c6 * s[28] + w0_c7 * s[29]);
s[32] = -1 * (s[30] * s[21]) + s[14] * s[31];
s[33] = w0_c5 * s[11] + (w0_c3 * s[12] + w0_c4 * s[13]);
s[34] = w0_c5 * s[27] + (w0_c3 * s[28] + w0_c4 * s[29]);
s[35] = -1 * (s[33] * s[31]) + s[34] * s[21];
s[36] = -1 * (s[14] * s[34]) + s[30] * s[33];
s[37] = s[26] * s[32] + (s[22] * s[35] + s[10] * s[36]);
s[52] = log(s[37]);
s[54] = 0.5 * (s[30] * s[31] + (s[22] * s[10] + s[14] * s[21]));
s[55] = pow(s[54], 2);
s[56] = 0.5 * (s[30] * s[34] + (s[22] * s[26] + s[14] * s[33]));
s[57] = 0.5 * (-1 + (pow(s[30], 2) + (pow(s[22], 2) + pow(s[14], 2))));
s[58] = 0.5 * (s[34] * s[31] + (s[26] * s[10] + s[33] * s[21]));
s[59] = 0.5 * (-1 + (pow(s[34], 2) + (pow(s[26], 2) + pow(s[33], 2))));
s[60] = 0.5 * (-1 + (pow(s[31], 2) + (pow(s[10], 2) + pow(s[21], 2))));
s[61] = exp(w[5][0] * (s[55] + (s[55] + 2 * pow(s[56], 2))) + (w[3][0] * pow(s[57], 2)
              + w[4][0] * (pow(s[58], 2) + (pow(s[59], 2) + pow(s[60], 2)))));
```

Topics

The uflacs project - what is working, what is not

Preliminary benchmark results

Short overview of algorithms

A key feature in FEniCS is the translation from symbolic equations to efficient low level code

- The symbolic equations are written in UFL code
- The translation is performed by the FEniCS Form Compiler
- FFC fails when the equations reach a certain complexity
- Uflacs is a project with new compiler algorithms to fix this

Uflacs can be installed today and used as a third representation in ffc

```
bzr branch lp:uflacs; cd uflacs
python setup.py install --prefix=/your/fenics/path
```

```python
from dolfin import *
# Use uflacs for everything:
parameters["form_compiler"]["representation"] = "uflacs"

# Or use uflacs for only this form:
p = {"representation":"uflacs"}
A = assemble(a, form_compiler_parameters=p)
```

```
ffc -r uflacs -l dolfin ffc/demo/HyperElasticity.ufl
g++ -c HyperElasticity.h
```

To reach full feature completeness with uflacs, there are a bunch of (mostly small) fixes left

- Integrals: dx, ds; dS, dP
- Expressions:
  almost everything;
  conditionals, jump, avg, higher order derivatives
- Geometry:
  x on cell, circumradius, facet normal, ...;
  x on facet
- Elements:
  full mixed element support;
  non-standard element mappings, quadrature elements

(This is obviously not a complete list).

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For a form compiler, there are three kinds of performace, all important

- Code generation time
- C++ compile time
- Assembly time

NB! The performance measurements presented next are done quickly as a reality check, this is still work in progress.

A basic hyperelastic model (see ffc demo)

```
# Copyright (C) 2009 Harish Narayanan
element = VectorElement("Lagrange", tetrahedron, 1)
v  = TestFunction(element)       # Test function
du = TrialFunction(element)      # Incremental displacement
u  = Coefficient(element)        # Previous displacement
B  = Coefficient(element)        # Body force per unit mass
T  = Coefficient(element)        # Traction force on boundary
F = Identity(3) + grad(u)        # Deformation gradient
C = F.T*F                        # Right Cauchy-Green tensor
E = variable((C-Identity(3))/2)  # Euler-Lagrange strain tensor
mu  = Constant(tetrahedron)      # Lame's constants
lam = Constant(tetrahedron)
psi = lam/2*(tr(E)**2) + mu*tr(E*E) # Strain energy function
S = diff(psi, E)        # Second Piola-Kirchhoff stress tensor
# The variational problem corresponding to hyperelasticity
L = inner(F*S, grad(v))*dx - inner(B, v)*dx - inner(T, v)*ds
a = derivative(L, u, du)
```

Comparing uflacs to quadrature representation for HyperElasticity.ufl – time to build

All numbers provided by ffc bench suite:

Representation	*Generate*	*Compile*	*Compile -O2*
uflacs	0.8 s	1.0 s	3 s
quadrature -O	12.9 s	1.6 s	5.1 s

Comparing uflacs to quadrature representation for HyperElasticity.ufl – time to compute (1)

All numbers provided by ffc bench suite:

Representation	*Generate*	*Compile*	*Compile -O2*
uflacs	0.8 s	1.0 s	3 s
quadrature -O	12.9 s	1.6 s	5.1 s

Runtime without -O2	*a*	*L*
uflacs	11.91 μs	4.25 μs
quadrature -O	9.37 μs	8.62 μs

Comparing uflacs to quadrature representation for HyperElasticity.ufl – time to compute (2)

All numbers provided by ffc bench suite:

Representation	*Generate*	*Compile*	*Compile -O2*
uflacs	0.8 s	1.0 s	3 s
quadrature -O	12.9 s	1.6 s	5.1 s

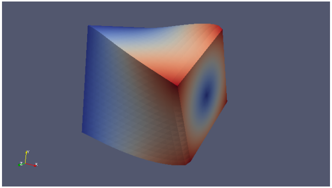
Runtime without -O2	*a*	*L*
uflacs	11.91 μs	4.25 μs
quadrature -O	9.37 μs	8.62 μs

Runtime with -O2	*a*	*L*
uflacs	2.72 μs	1.10 μs
quadrature -O	2.65 μs	2.65 μs

Uflacs provides twice as fast assembly in dolfin hyperelasticity demo

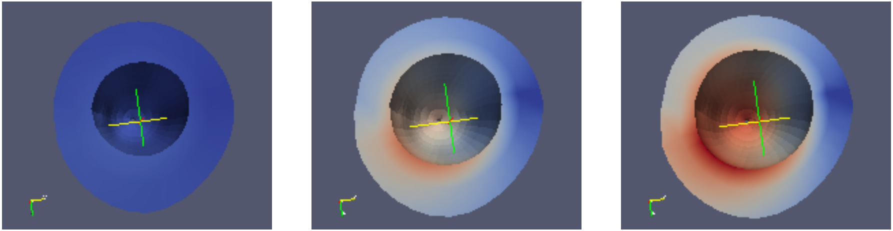
Assemble cells	*Average time*
uflacs	0.27 s
quadrature	0.55 s

Assemble facets	*Average time*
uflacs	0.02295
quadrature	0.02252



Numbers provided by timings().

Uflacs enables new applications in FEniCS: Here large deformation of a left ventricle with anisotropic hyperelastic material



An excerpt of a Fung type anisotropic hyperelasticity model – previously not feasible in FEniCS

```
# Identity matrix and global deformation gradient
F_glob = I + grad(u)
F = variable(R.T*F_glob*R)
E = 0.5*(F.T*F - I)
J = det(F)
# Fung-type material law
f=0; s=1; n=2
W = (bff*E[f,f]**2 + bxx*(E[n,n]**2 + E[s,s]**2 + E[n,s]**2)
     + bfx*(E[f,n]**2 + E[n,f]**2 + E[f,s]**2 + E[s,f]**2))
psi = 0.5*K*(exp(W) - 1) + Ccompr*(J*ln(J) - J + 1)
P = R*diff(psi, F)*R.T # First Piola-Kirchoff stress tensor
# Neumann boundary condition
sigma = Constant(-0.02)
T = dot(det(F_glob)*sigma*inv(F_glob.T), N)
```

Time to jit and assemble matrix for Poisson compared to Fung type anisotropic hyperelasticity

assemble(a)	tensor/P	quadr/P	uflacs/P	uflacs/Fung
Clean cache	2.367 s	2.506 s	2.452 s	7.077 s
Memory cache	0.045 s	0.068 s	0.218 s	0.568 s
Disk cache	0.049 s	0.067 s	0.216 s	1.644 s
Memory cache	0.045 s	0.062 s	0.213 s	0.567 s

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UFL represents symbolic expressions as a Directed Acyclic Graph (DAG)

- Each node is represented by a subclass of Terminal or Operator
- Each node can be tensor valued
- Some operators represent computation (e.g. addition)
- Other operators represent only reshaping (e.g. indexing)

UFLACS was designed for tensor intensive equations – that make heavy use of tensor algebra features in UFL

- Algorithms produce in a lot of symbolic patterns similar to *indexing* $\rightarrow$ *scalar operators* $\rightarrow$ *indexed-to-tensor*
- Operations such as `A[i,j,k]`, `as_tensor(A[i,j,k],(k,i,j))`, and `A.T` should not contribute to computations but increase symbolic complexity
- Uflacs algorithms were designed with this in mind

The initial stages of the uflacs compiler algorithm

- Translate the DAG from node-based to list-based representation
- Apply value numbering of each scalar subexpression component involving a computation
- Value numbering “falls through” reshaping type operators

After the initial stages, the expression has been translated to a list of scalar expressions

- Each subexpression is either
  - a scalar operator performing some computation, or
  - a *modified terminal*
- *Modified terminals* are terminals with eventual grad, restriction, and indexed operators applied
- A modified terminal represents a scalar expression that uflacs does not know how to compute *(needs geometry or elements)*

In the intermediate stages, dependencies are represented and analysed using integer arrays

- Easy with array based DAG storage with scalar nodes
- Edges are therefore efficient to invert and count
- Only modified terminals that are referenced by operator nodes are stored
- Edge arrays are used to e.g.
  - Decide loop placement of subexpressions
  - Prioritize intermediate variable storage of subexpressions
  - (Quite crude algorithms at this stage)

In the code generation stage, a generic code generator delegates modified terminals to a backend

- A generic compiler routine in uflacs produces C(++) code with backend-specific code inserted on demand
- An ffc backend in uflacs generates code to compute modified terminals based tables of element basis function values passed from FFC
- A dolfin backend in uflacs generates a `dolfin::Expression` subclass, including code to evaluate a `GenericFunction` member inside the `Expression::eval` implementation

Current state of ffc-uflacs project relations (it's not as messy as it may sound...)

- ffc uses ffc.uflacsrepr to generate `tabulate_tensor`
- ffc.uflacsrepr delegates most of the work to uflacs.backends.ffc
- uflacs.backends.ffc uses the generic uflacs.algorithms.compiler to do most of the work, passing it callbacks to generate code for computing modified terminals (geometry and functions)

Questions?

- Try uflacs on your forms at the “Ask the developer” session later today!
- Report bugs to *http://bugs.launchpad.net/uflacs*
- If you have a form that still takes long to build, send it to me and I can use it for profiling later.
- *martinal@simula.no*