Shear Banding in the Earth's Mantle

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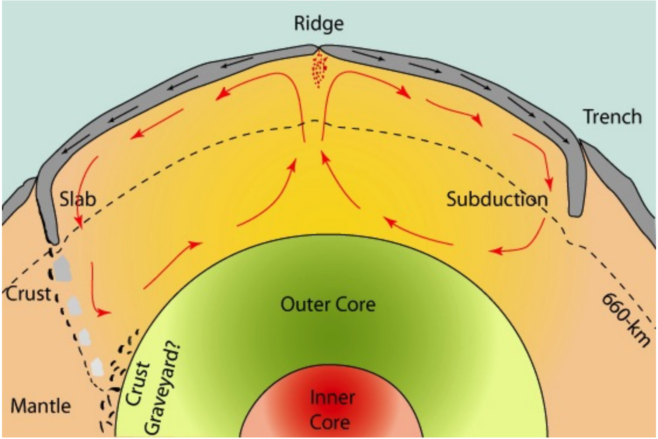
Mantle convection

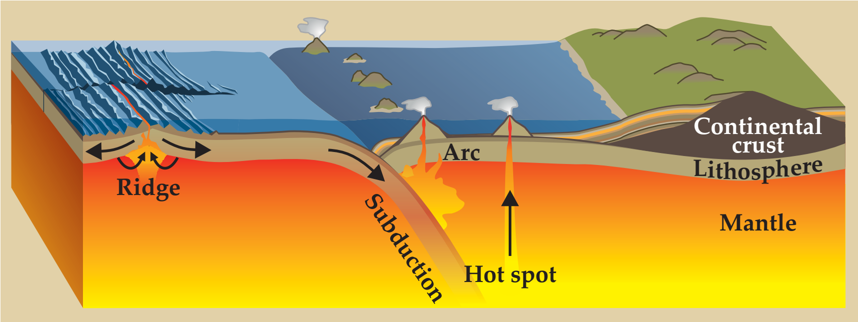
Hot fluid mantle is heated from below, cooled at the top
Convection drives cold stiff plates → Coupled system



[U. Alberta]

Ridges and subduction zones

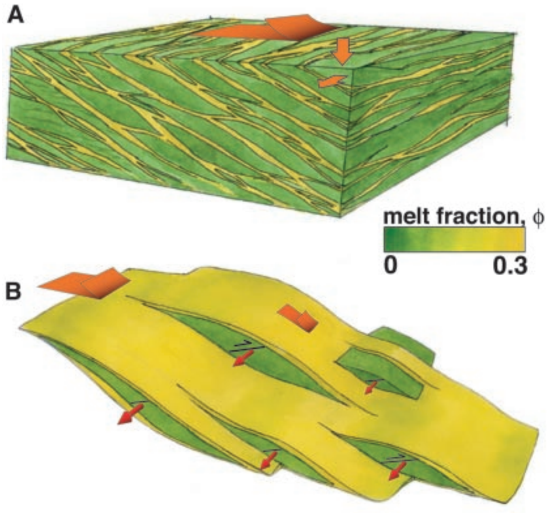
- Plates created at mid-oceanic ridges, move towards trenches, recycled in subduction zones
- Mantle properties determine plate motion



[Hirschmann & Kohlstedt, 2012]

Mantle-magma interaction important in subduction zones: melting in mantle wedge, formation of island arcs

Zooming in: convection and compaction



[Holtzman et al, 2003]

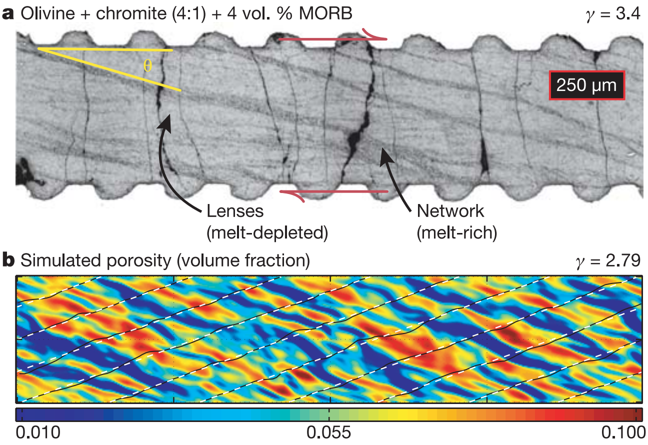
Deformation processes on mm scale influence large-scale features

Mantle is partially molten $\rightarrow$ flow of magma through compacting and convecting porous matrix

Shear causes melt to segregate $\rightarrow$ shear bands $\rightarrow$ mechanism for larger-scale melt transport

Zooming in: convection and compaction

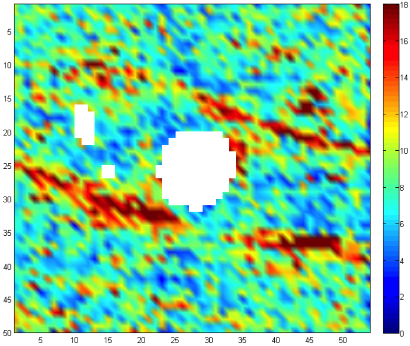
Compare numerical models with shear banding in laboratory experiments $\rightarrow$ material properties?



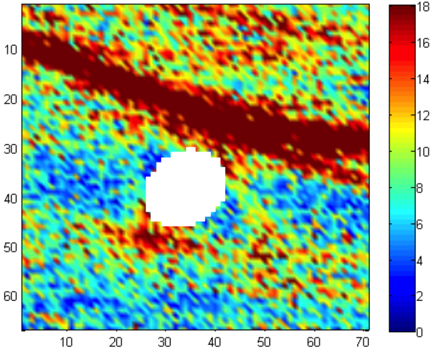
[Katz et al, 2006]

Inclusion in porous medium under simple shear

Melt mapping in laboratory experiment: olivine + 10% MORB



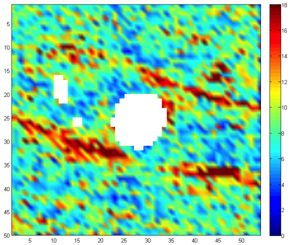
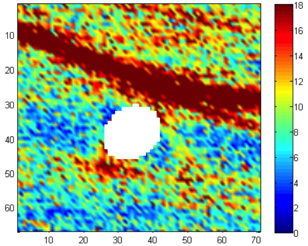
$\gamma = 1.0$



$\gamma = 2.0$

[Chao Qi & David Kohlstedt]

Inclusion in porous medium under simple shear

- Is formation of shear bands dominant over compaction around the inclusion?
- What determines this balance?
- Is there asymmetry between melt enrichment and depletion?
- What affects this asymmetry?

$\rightarrow$ *nonlinearity, viscosity ratios, total strain*

Equations: Compaction and advection

Conservation of mass for the solid phase:

$$\frac{\partial \phi}{\partial t} + \mathbf{v_s} \cdot \nabla \phi = (1 - \phi) \nabla \cdot \mathbf{v_s} + \frac{\Gamma}{\rho_s} \tag{1}$$

Conservation of mass for the two-phase mixture:

$$\nabla \cdot \overline{\mathbf{v}} + \Gamma \Delta \left(\frac{1}{\rho} \right) = 0 \tag{2}$$

Conservation of momentum for the fluid:

$$\nabla \cdot (\phi \boldsymbol{\sigma_f}) + \phi \rho_f \mathbf{g} - \mathbf{F} = \mathbf{0} \tag{3}$$

Conservation of momentum for the solid:

$$\nabla \cdot ((1 - \phi) \boldsymbol{\sigma_s}) + (1 - \phi) \rho_s \mathbf{g} + \mathbf{F} = \mathbf{0} \tag{4}$$

Equations

Compaction and advection simplified:

$$\frac{\partial \phi}{\partial t} + \mathbf{v_s} \cdot \nabla \phi - (1-\phi) \nabla \cdot \mathbf{v_s} = 0 \quad (5)$$

$$\nabla \cdot \left(-\frac{K_\phi}{\mu_f} \nabla P + \mathbf{v_s} \right) = 0 \quad (6)$$

$$\nabla P = \nabla \cdot \big(\eta_\phi (\nabla \mathbf{v_s} + \nabla \mathbf{v_s}^T) \big) + \nabla \cdot \left((\zeta_\phi - \frac{2}{3} \eta_\phi) \nabla \cdot \mathbf{v_s} \right) \quad (7)$$

[after McKenzie, 1984]

Porosity-dependent rheology

Permeability

$$K_\phi = \phi^2 \tag{8}$$

Bulk viscosity

$$\zeta_\phi = \frac{1}{\phi} \tag{9}$$

Shear viscosity

$$\eta_\phi = \eta_0 \; e^{-\alpha(\phi - \phi_0)} \tag{10}$$

Compaction length

$$\delta_c = \sqrt{\frac{K_0}{\mu_f}\left(\zeta_0 + \frac{4}{3}\eta_0\right)} \tag{11}$$

Benchmark 1: Compaction around sphere

Analytical solution

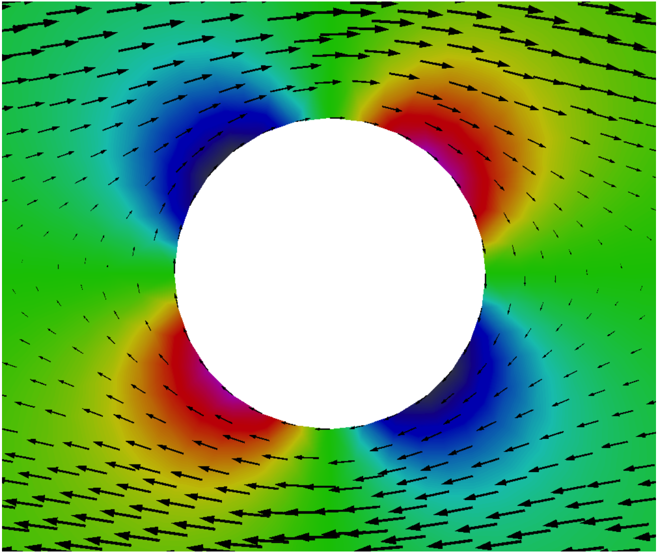
$$\mathbf{v}_s = \left(-\frac{4D}{r^4} + \frac{2FK_2(r)}{r^2}\right)\underline{\underline{\mathbf{E}}}\cdot\mathbf{x} + \left(-\frac{2C}{r^4} + \frac{8D}{r^6} - \frac{FK_3(r)}{r^3}\right)(\mathbf{x}\cdot\underline{\underline{\mathbf{E}}}\cdot\mathbf{x})\mathbf{x} \tag{12}$$

$$C = -\frac{a^4 K_2'(a)}{4\xi K_1(a) - a^2 K_2'(a)}, \tag{13}$$

$$D = \frac{a^4}{4} + \frac{4a^3\xi K_2(a)}{4\xi K_1(a) - a^2 K_2'(a)}, \tag{14}$$

$$F = \frac{8a\xi}{4\xi K_1(a) - a^2 K_2'(a)}, \tag{15}$$

Benchmark 1: Compaction around sphere



Benchmark 2: Plane wave

Initial condition

$$\phi_i(x_i, y_i) = 1.0 + A\cos\left(k_0 x_i \sin(\theta_0) + k_0 y_i \cos(\theta_0)\right) \tag{16}$$

Analytical growth rate of planar shear bands

$$\dot{s}_a = -2\alpha\xi \frac{(1-\phi_0)}{\phi_0} \frac{k_x k_y}{k^2+1} \tag{17}$$

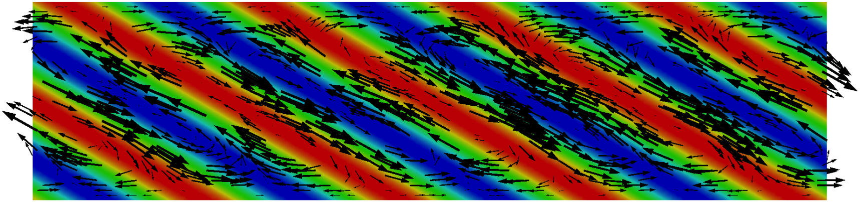
Numerical growth rate

$$\dot{s}_n = \frac{(1-\phi_0)}{\phi_0 A} \, \nabla \cdot \mathbf{v_s} \tag{18}$$

[Spiegelman, 2003]

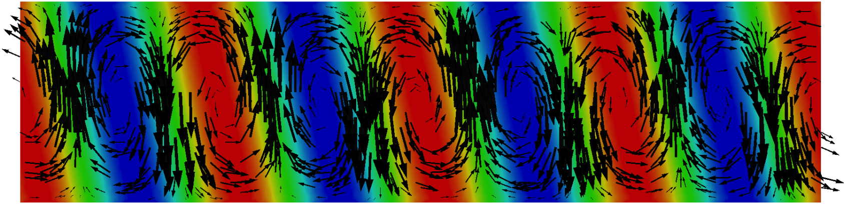
Benchmark 2: Plane wave

Porosity and velocity perturbation at $\gamma = 0$



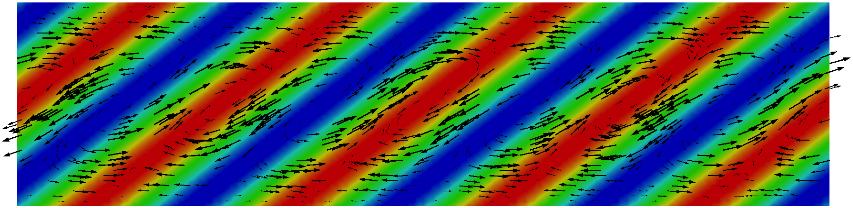
Benchmark 2: Plane wave

Porosity and velocity perturbation at $\gamma = 1.5$

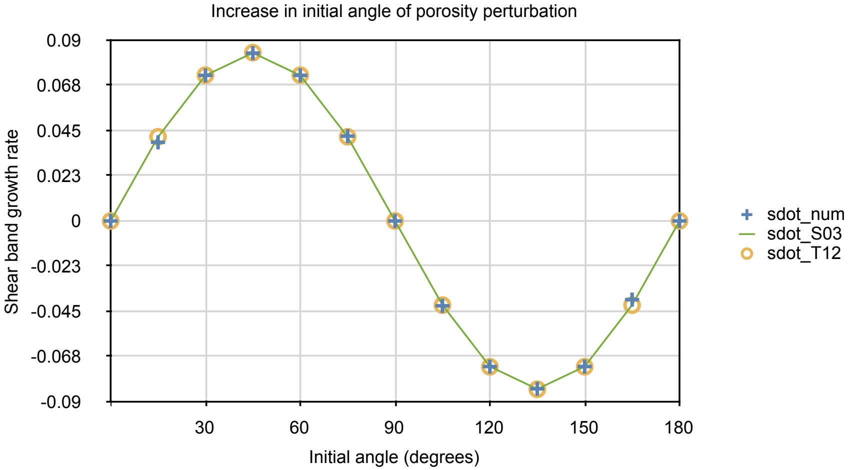


Benchmark 2: Plane wave

Porosity and velocity perturbation at $\gamma = 3.0$

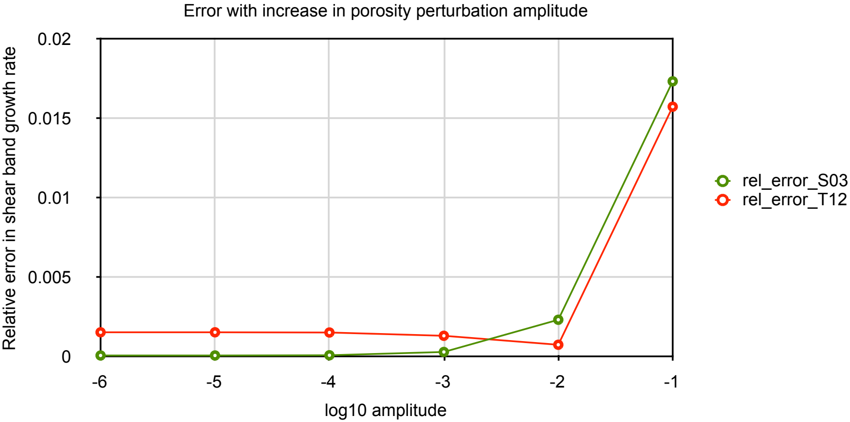


Benchmark 2: Initial angle



- Growth rate depends on initial shear band angle
- Fit analytical rates well

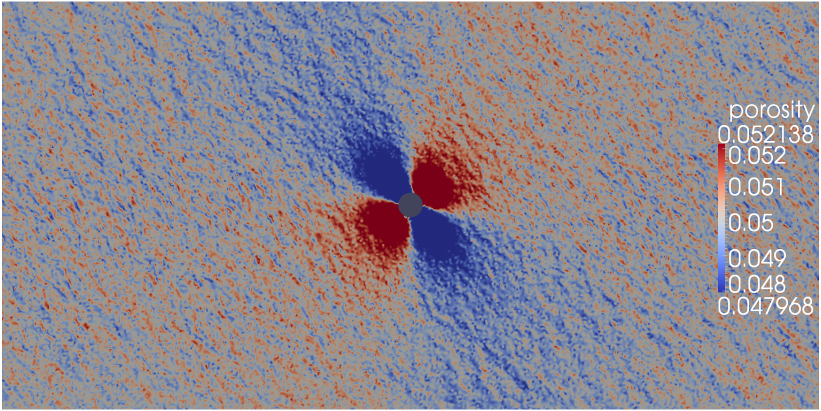
Benchmark 2: Perturbation amplitude



- Error increases for increasing perturbation amplitude
- Small perturbation assumption breaks down $\gtrsim 10^{-2}$

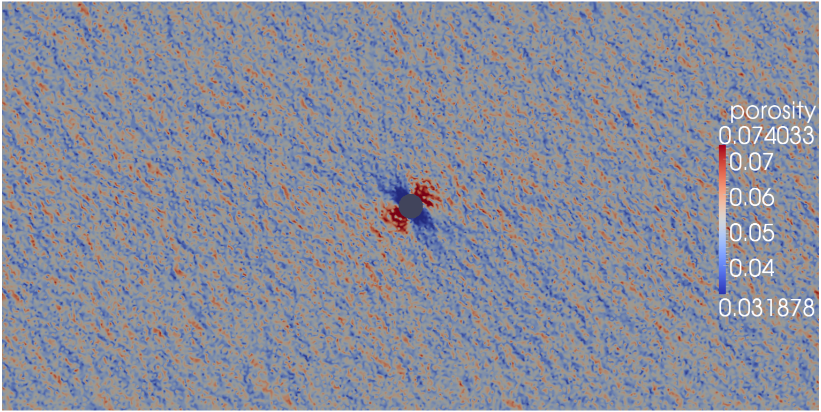
Pressure shadows and shear bands

Initial porosity perturbation amplitude 10^{-3}



Pressure shadows and shear bands

Initial porosity perturbation amplitude 10^{-2}



Pressure shadows and shear bands

What affects relative importance?

- Nonlinearity of porosity dependence α
- Ratio of bulk to shear viscosity ζ_0/η_0
- Amplitude of initial perturbation A