FFC User Manual

June 27, 2005

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`http://www.fenics.org/ffc/`

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1 Introduction

This is a first draft for a manual for FFC manual. Contributions are most welcome.

2 Installation

In preparation.

A Reference elements

A.1 The reference triangle

The reference triangle (Figure 1) is defined by the following three vertices:

$$\begin{aligned} v^0 &= (0, 0), \\ v^1 &= (1, 0), \\ v^2 &= (0, 1). \end{aligned} \tag{1}$$

Note that this corresponds to a counter-clockwise orientation of the vertices in the plane.

The edges of the reference triangle are ordered following the convention that edge e^i should be opposite to vertex v^i for $i = 0, 1, 2$, with the vertices of each edge ordered to give a counter-clockwise orientation of the triangle in the plane:

$$\begin{aligned} e^0 &: (v^1, v^2), \\ e^1 &: (v^2, v^0), \\ e^2 &: (v^0, v^1). \end{aligned} \tag{2}$$

A.2 The reference tetrahedron

The reference tetrahedron (Figure 3) is defined by the following four vertices:

$$\begin{aligned} v^0 &= (0, 0, 0), \\ v^1 &= (1, 0, 0), \\ v^2 &= (0, 1, 0), \\ v^4 &= (0, 0, 1). \end{aligned} \tag{3}$$

The faces of the reference tetrahedron are ordered following the convention that face f^i should be opposite to vertex v^i for $i = 0, 1, 2, 3$, with the vertices of each face ordered to give a counter-clockwise orientation of each face as

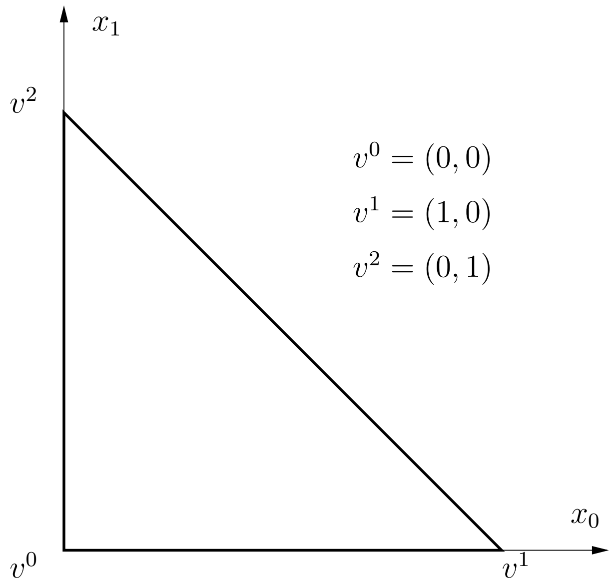

Figure 1: Physical coordinates of the reference triangle.

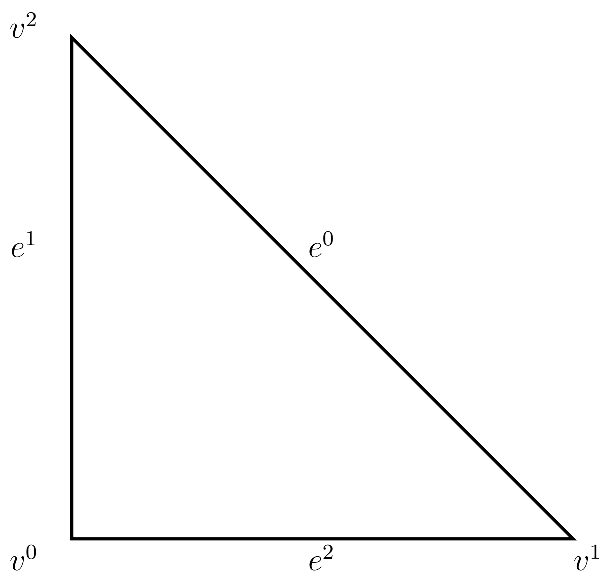

Figure 2: Ordering of mesh entities (vertices and edges) for the reference triangle.

seen from the outside of the tetrahedron and the first vertex of face f^i given by vertex $v^{i+1 \mod 4}$:

$$\begin{aligned}
f^0 &: (v^1, v^3, v^2), \\
f^1 &: (v^2, v^3, v^0), \\
f^2 &: (v^3, v^1, v^0), \\
f^3 &: (v^0, v^1, v^2).
\end{aligned} \tag{4}$$

The edges of the reference tetrahedron are ordered following the convention that edges e^0, e^1, e^2 should correspond to the edges of the reference triangle. Edges e^3, e^4, e^5 all ending up at vertex v^3 are ordered based on their first vertex:

$$\begin{aligned}
e^0 &: (v^1, v^2), \\
e^1 &: (v^2, v^0), \\
e^2 &: (v^0, v^1), \\
e^3 &: (v^0, v^3), \\
e^4 &: (v^1, v^3), \\
e^5 &: (v^2, v^3).
\end{aligned} \tag{5}$$

The ordering of vertices on faces implicitly defines an ordering of edges on faces by identifying an edge on a face with the opposite vertex on the face:

$$\begin{aligned}
f^0 &: (e^5, e^0, e^4), \\
f^1 &: (e^3, e^1, e^5), \\
f^2 &: (e^2, e^3, e^4), \\
f^3 &: (e^0, e^1, e^2).
\end{aligned} \tag{6}$$

Note that the ordering of edges on f^3 is the same as the ordering of edges on the reference triangle. Also note that the internal ordering of vertices on edges does not always follow the orientation of the face (which is not possible).

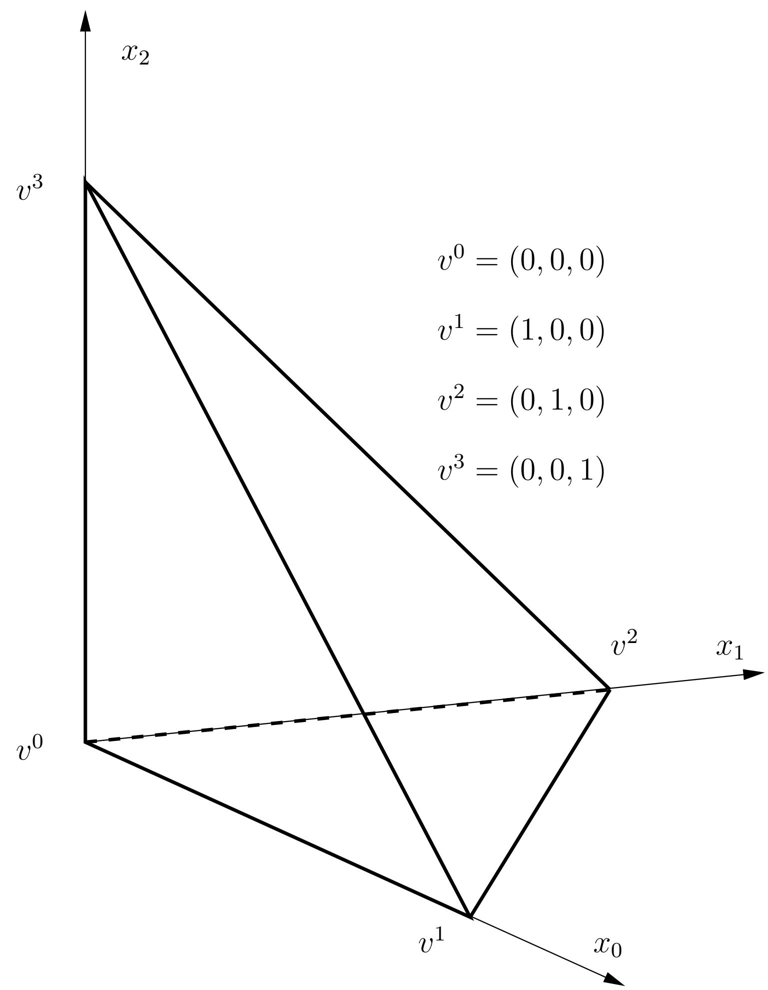

Figure 3: Physical coordinates of the reference tetrahedron.

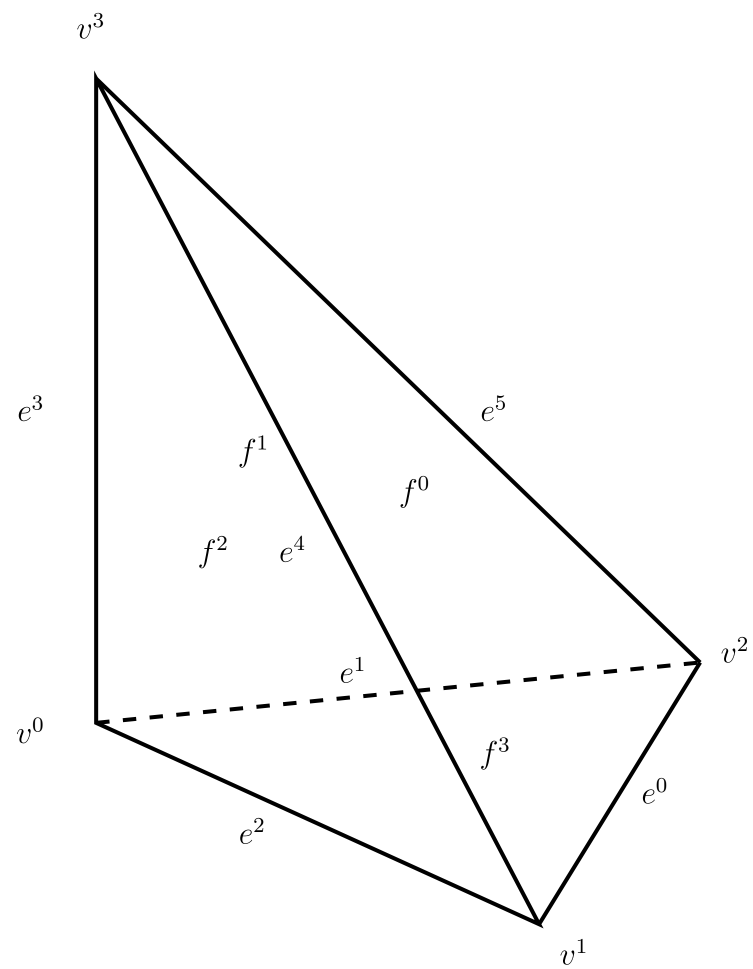

Figure 4: Ordering of mesh entities (vertices, edges, faces) for the reference tetrahedron.

B Ordering of degrees of freedom

The local and global orderings of degrees of freedom or *nodes* are obtained by associating each node with a mesh entity, locally and globally.

B.1 Mesh entities

We distinguish between mesh entities of different topological dimensions:

vertices	topological dimension 0
edges	topological dimension 1
faces	topological dimension 2
cells	topological dimension 2 or 3

A cell can be either a triangle or a tetrahedron depending on the type of mesh. For a mesh consisting of triangles, the mesh entities involved are vertices, edges and cells, and for a mesh consisting of tetrahedrons, the mesh entities involved are vertices, edges, faces and cells.

B.2 Ordering among mesh entities

With each mesh entity, there can be associated zero or more nodes and the nodes are ordered locally and globally based on the topological dimension of the mesh entity with which they are associated. Thus, any nodes associated with vertices are ordered first and nodes associated with cells last.

If more than one node is associated with a single mesh entity, the internal ordering of the nodes associated with the mesh entity becomes important, in particular for edges and faces, where the nodes of two adjacent cells sharing a common edge or face must lign up.

B.3 Internal ordering on edges

For edges containing more than one node, the nodes are ordered in the direction from the first vertex (v_e^0) of the edge to the second vertex (v_e^1) of the edge as in Figure 5.

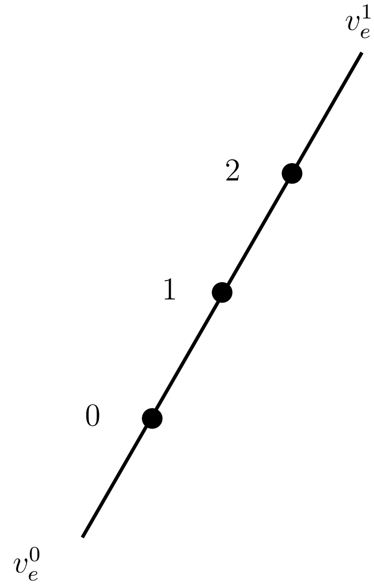

Figure 5: Internal ordering of nodes on edges.

B.4 Alignment of edges

Depending on the orientation of any given cell, an edge on the cell may be aligned or not aligned with the corresponding edge on the reference cell if the vertices of the cell are mapped to the reference cell. We define the *alignment* of an edge with respect to a cell to be 0 if the edge is aligned with the orientation of the reference cell and 1 otherwise.

Example 1: The alignment of the first edge (e^0) on a triangle is 0 if the first vertex of the edge is the second vertex (v^1) of the triangle.

Example 2: The alignment of the second edge (e^1) on a tetrahedron is 0 if the first vertex of the edge is the third vertex (v^2) of the tetrahedron.

If two cells share a common edge and the edge is aligned with one of the cells and not the other, we must reverse the order in which the local nodes are mapped to global nodes on one of the two cells. As a convention, the order is kept if the alignment is 0 and reversed if the alignment is 1.

B.5 Internal ordering on faces

For faces containing more than one node, the ordering of nodes is nested going from the first to the third vertex and in each step going from the first to the second vertex as in Figure 6.

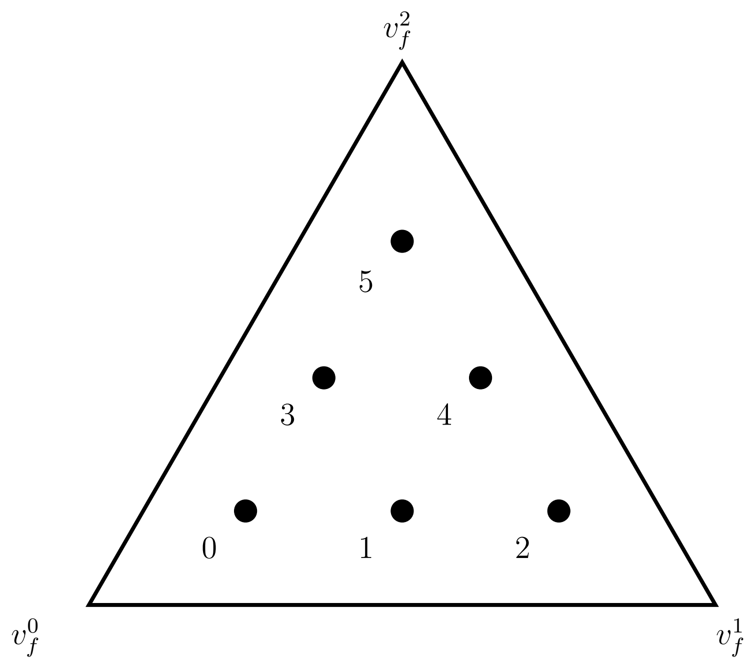

Figure 6: Internal ordering of nodes on faces.

B.6 Alignment of faces

There are six different ways for a face to be aligned on a tetrahedron; there are three ways to pick the first edge of the face, and once the first edge is picked, there are two ways to pick the second edge. To define an alignment of faces as an integer between 0 and 5, we compare the ordering of edges on a face with the ordering of edges on the corresponding face on the reference tetrahedron. If the first edge of the face matches the first edge on the corresponding face on the reference tetrahedron and also the second edge matches the second edge on the reference tetrahedron, then the alignment is 0. If only the first edge matches, then the alignment is 1. We similarly define alignments $2, 3$ by matching the first and second edges with the second and third edges on the corresponding face on the reference tetrahedron, and alignments $4, 5$ by matching the first and second edges with the third and first edges on the corresponding face on the reference tetrahedron.

Example 1: The alignment of the first face of a tetrahedron is 0 if the first edge of the face is edge number 5 and the second edge is edge number 0.

Example 2: The alignment of the first face of a tetrahedron is 1 if the first edge of the face is edge number 5 and the second edge is not edge number 0. (It must then be edge number 4.)

Example 3: The alignment of the first face of a tetrahedron is 4 if the first edge of the face is edge number 4 and the second edge is edge number 5.

Example 4: The alignment of the first face of a tetrahedron is 5 if the first edge of the face is edge number 4 and the second edge is not edge number 5. (It must then be edge number 0.)

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