Automation of Turbulence Simulation

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Some References

* J.Hoffman, Computation of mean drag for bluff body problems using Adaptive DNS/LES, *SIAM J. Sci. Comput. Vol.27(1)*, 2005.

* J.Hoffman and C.Johnson, A new approach to Computational Turbulence Modeling, to appear in *Comput. Methods Appl. Mech. Engrg.*, 2005.

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* J.Hoffman, Efficient computation of mean drag for the subcritical flow past a circular cylinder using Adaptive DNS/LES, in review (*Int. J. of Numerical Methods in Fluids*) .

* J.Hoffman, Adaptive simulation of the turbulent flow due to a cylinder rolling along ground, in review (*Comput. Methods Appl. Mech. Engrg.*).

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Automation of Turbulence Simulation

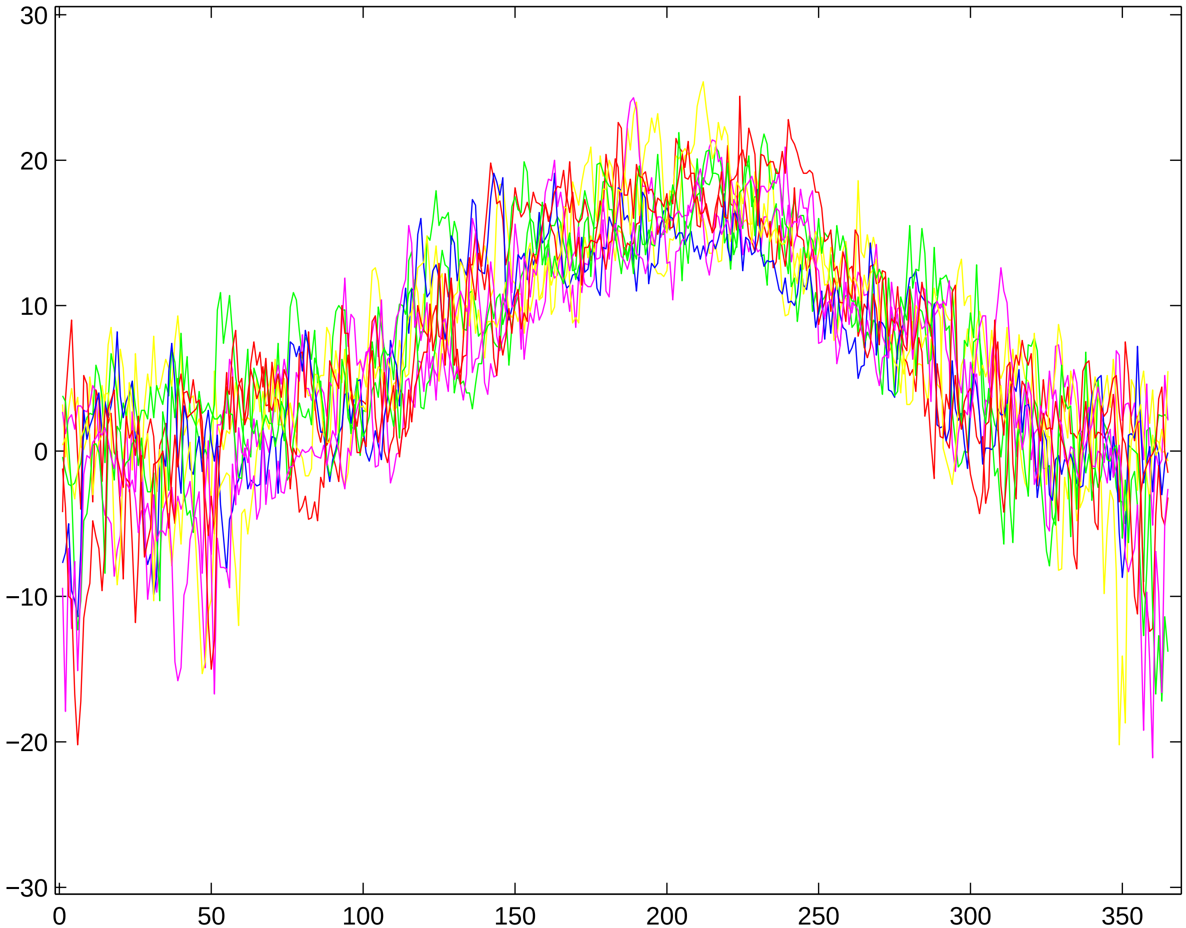
- ACMM: Generality in: Method and Implementation
- Method: Adaptive stabilized FEM: General Galerkin G2
- Implementation: FEniCS project

This talk: Test of Method: G2

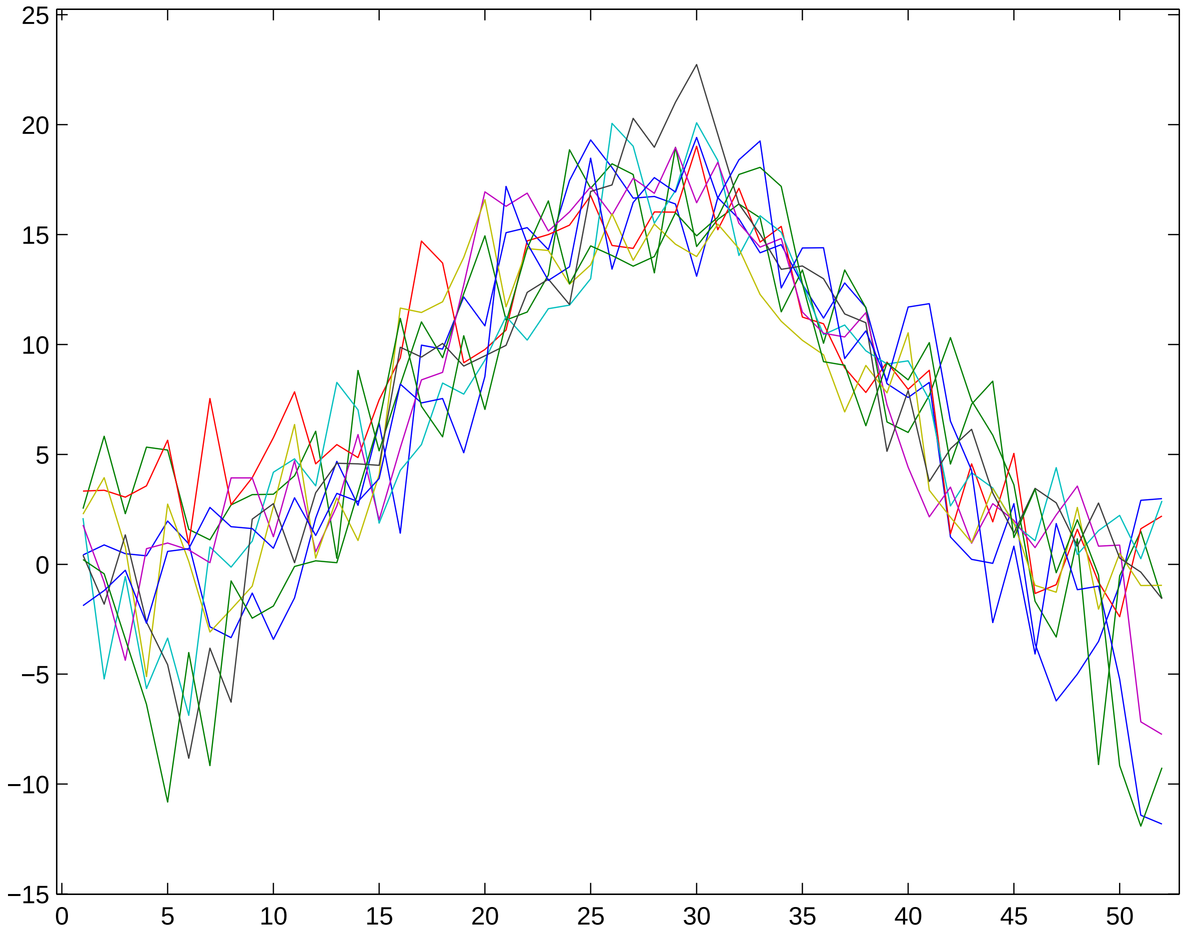
Turbulence Simulation; characterized by non-generality:

- discretization (geometry specific FD, spectral,...)
- turbulence modeling (parameters depend on data, numerics,...; different filters used in RANS/LES,...)
- wall modeling using RANS, boundary layer theory,...

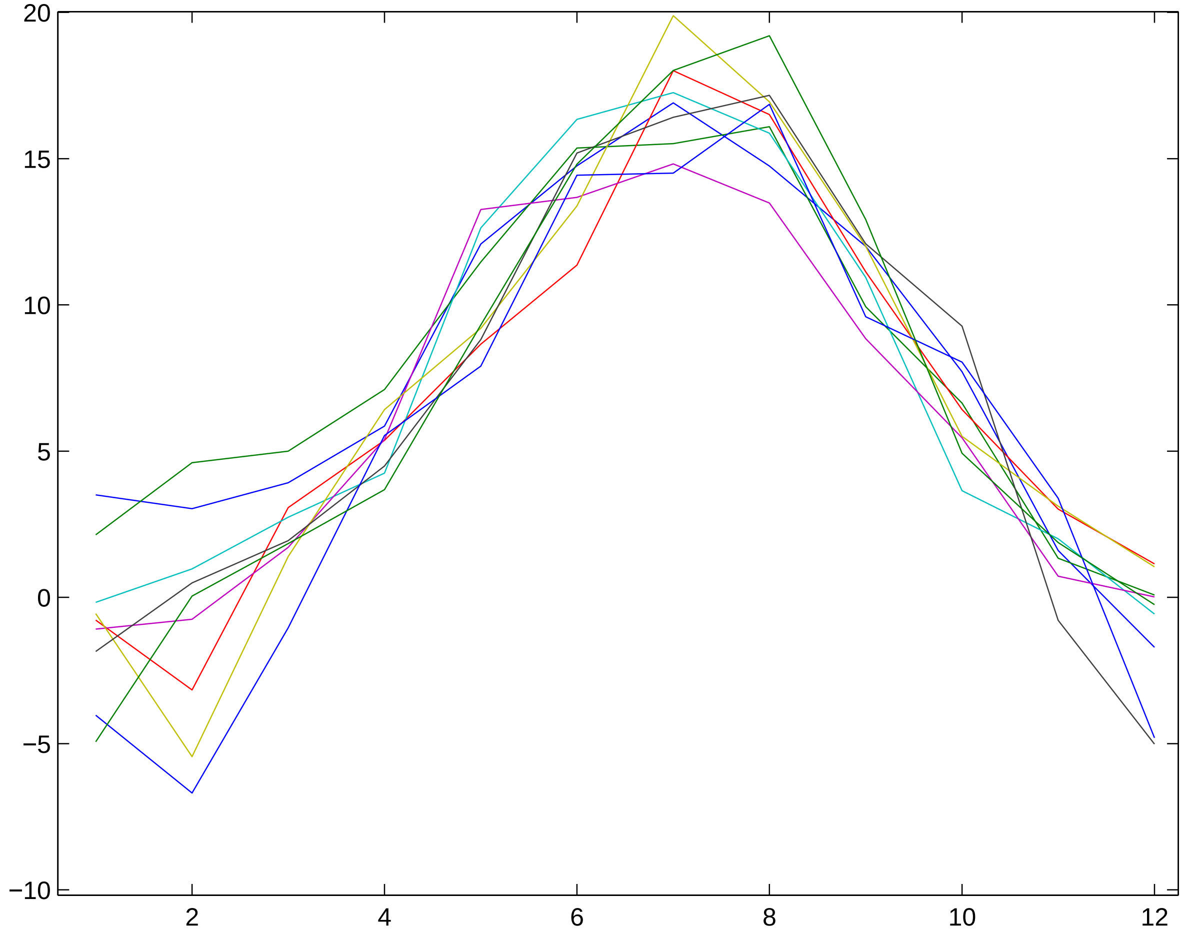
Målilla 88-95: 24-hr mean: $\pm 10°C$



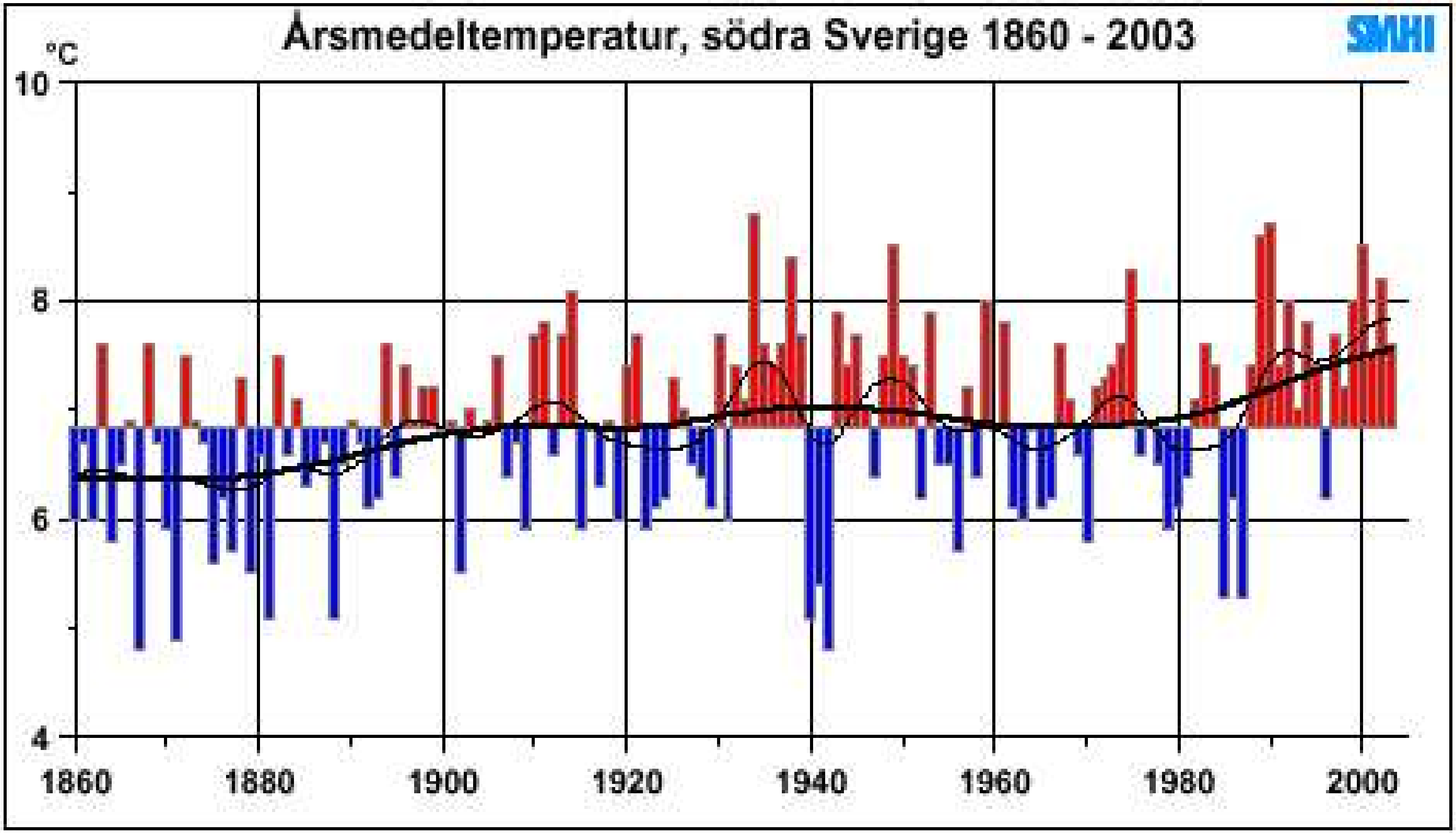
Målilla 88-95: weekly mean: $\pm 5°C$



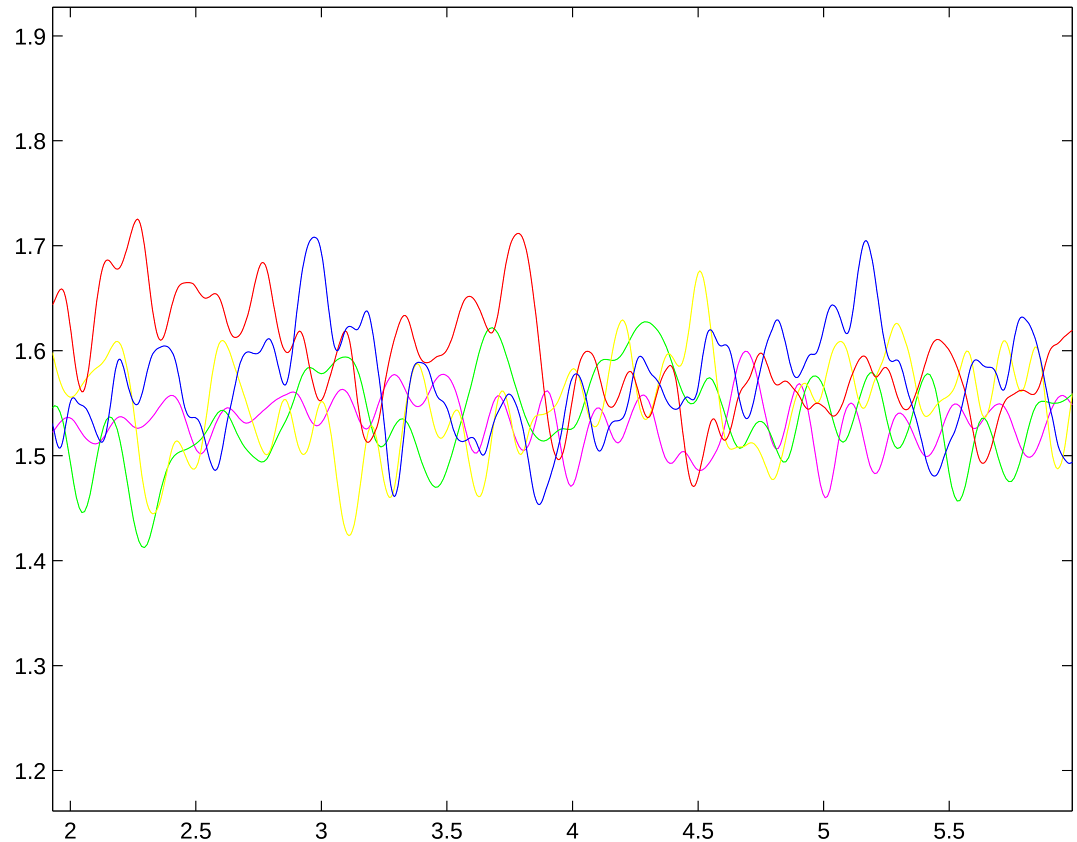
Målilla 88-95: monthly mean: $\pm 3°C$



Yearly mean 1860-2003: $\pm 1°C$



Cube: $c_D(t) \pm 10\%$, mean $c_D \pm 2\%$



cube: xy-plane *cube: xz-plane*

Turbulence: Chaotic Dyn. System

- Pointwise quantities strongly sensitive to perturbations and thus unpredictable (to any tolerance of interest)
- Mean values in space/time moderately sensitive to perturbations and thus predictable up to a tolerance
- Ex: Weather prediction (Målilla):
  24-hr mean temperature unpredictable ($\pm 10°$C)
  monthly mean temperature predictable ($\pm 3°$C)
- Ex: Bluff body flow (experiments/computations):
  pointwise drag $c_D(t)$ unpredictable ($> \pm 10\%$)
  mean value drag $\int_I c_D(t)\, dt$ predictable ($\pm 2\%$)

Navier-Stokes Equations (NSE)

$$R(\hat{u}) = \begin{pmatrix} \dot{u} + u \cdot \nabla u - \nu \Delta u + \nabla p - f \\ \nabla \cdot u \end{pmatrix} = 0$$

$\hat{u} = (u, p)$: u velocity, p pressure, ν viscosity, $f = 0$

- Pointwise existence & uniqueness unknown: $R(\hat{u}) = 0$ (Clay Institute $1 million Prize Problem)
- Existence (but not uniq.) of weak solution (Leray 1934):

Find $\hat{u} \in \hat{V}$: $(R(\hat{u}), \hat{v}) = 0 \quad \forall \hat{v} = (v, q) \in \hat{V}$

$\hat{V} \subset H^1(Q)$: $Q = \Omega \times I$ space-time domain, $(\cdot,\cdot) = (\cdot,\cdot)_Q$

$$(R(\hat{u}), \hat{v}) \equiv (\dot{u}, v) + (u \cdot \nabla u, v) - (\nabla \cdot v, p) + (\nabla \cdot u, q) + (\nu \nabla u, \nabla v)$$

Navier-Stokes Equations (NSE)

$$R(\hat{u}) = \begin{pmatrix} \dot{u} + u \cdot \nabla u - \nu \Delta u + \nabla p - f \\ \nabla \cdot u \end{pmatrix} = 0$$

$\hat{u} = (u, p)$: u velocity, p pressure, ν viscosity, $f = 0$

- Computational turbulence: Mean values computable ($< \pm 5\%$), point values not computable ($> \pm 10\%$).
- Computational cost (#dofs): $\sim Re^3 \quad (Re = UL/\nu)$ (resolving all scales in Direct Num. Simulation DNS)

  Limit today: $Re \approx 10^3$ (many industrial appl. $Re > 10^6$)

Analysis & Computation of NSE

Scientific goals today:

- Prove exist & uniq of pointwise NSE: $R(\hat{u}) = 0$
- Push limit of DNS (wrt Re^3 constraint)
- Turbulence modeling: find model for unresolved scales (filtering of NSE: Reynolds stresses, closure problem)

Alternative scientific goals:

- Approximate weak solution $\hat{U}$: weak uniqueness in (mean value) output $M(\hat{U})$: stability of $\hat{U}$ wrt $M(\cdot)$
- Adaptive algorithm: min #dof : $error(M(\hat{U})) < TOL$

Existence of ϵ-Weak Solutions

- $W_\epsilon = \{\hat{u} \in \hat{V} : |(R(\hat{u}), \hat{v})| \leq \epsilon \|\hat{v}\|_{\hat{V}} \quad \forall \hat{v} \in \hat{V}\}$

  (approximate weak solution: $\sim \|R(\hat{u})\|_{H^{-1}} \leq \epsilon$)
- Existence: Construction of W_ϵ: General Galerkin G2
- Find $\hat{U} \in \hat{V}_h$: $\quad (R(\hat{U}), \hat{v}) + (hR(\hat{U}), R(\hat{v})) = 0 \quad \forall \hat{v} \in \hat{V}_h$

$$\frac{1}{2}\|U(t)\|^2 + \|\sqrt{\nu}\nabla U\|_Q^2 + \|\sqrt{h}R(\hat{U})\|_Q^2 = \frac{1}{2}\|U(0)\|^2$$

$$\begin{aligned}(R(\hat{U}), \hat{v}) &= (R(\hat{U}), \hat{v} - \pi_h \hat{v}) - (hR(\hat{U}), R(\pi_h \hat{v})) \\ &\leq (C + M_U)\, \|hR(\hat{U})\|_Q\, \|\hat{v}\|_{\hat{V}} \leq C\sqrt{h}\, \|\hat{v}\|_{\hat{V}}\end{aligned}$$

- $\hat{U} \in$ G2 $\Rightarrow \hat{U} \in W_\epsilon \qquad \epsilon = (C + M_U)\, \|hR(\hat{U})\|_Q$

Weak Uniqueness: Duality

- Output (functional): $M(\hat{u}) \equiv (\hat{u}, \hat{\psi})$
- Dual NSE: Find $\hat{\varphi} = (\varphi, \theta)$: $\quad a(\hat{u}, \hat{w}; \hat{v}, \hat{\varphi}) = M(\hat{v}) \;\; \forall \hat{v} \in \hat{V}_0$
  $v = (v, q) \in \hat{V}_0 \quad \hat{V}_0 = \{\hat{v} \in \hat{V} : v(\cdot, 0) = 0\}$

$$
\begin{aligned}
a(\hat{u}, \hat{w}; \hat{v}, \hat{\varphi}) \equiv (\dot{v}, \varphi) + (u \cdot \nabla v, \varphi) + (v \cdot \nabla w, \varphi) \\
- (\nabla \cdot \varphi, q) + (\nabla \cdot v, \theta) + (\nu \nabla v, \nabla \varphi)
\end{aligned}
$$

- Weak Uniqueness: $\hat{u}, \hat{w} \in W_\epsilon, \quad S_\epsilon(\hat{\psi}) \equiv \max_{\hat{u}, \hat{w} \in W_\epsilon} ||\hat{\varphi}||_{\hat{V}}$

$$
\begin{aligned}
|M(\hat{u}) - M(\hat{w})| &= |a(\hat{u}, \hat{w}; \hat{u} - \hat{w}, \hat{\varphi})| \\
&= |(R(\hat{u}), \hat{\varphi}) - (R(\hat{w}), \hat{\varphi})| \leq 2\epsilon S_\epsilon(\hat{\psi})
\end{aligned}
$$

- Exact solution ($\epsilon = 0$): stability information lost!

Weak Uniqueness: Duality

- Weak Uniqueness: $\hat{u}, \hat{w} \in W_\epsilon$

$$|M(\hat{u}) - M(\hat{w})| \leq 2\epsilon S_\epsilon(\hat{\psi}) \quad \|R(\hat{u})\|_{H^{-1}}, \|R(\hat{w})\|_{H^{-1}} \leq \epsilon$$

- Computability: $\hat{u} \in W_\epsilon$ and $\hat{U} \in \; G2$

$$|M(\hat{u}) - M(\hat{U})| \;\leq\; (\epsilon + \epsilon_{G2})\; S_{\epsilon_{G2}}(\hat{\psi}) \quad \epsilon_{G2} = C\|hR(\hat{U})\|$$

- Residual only needs to be small in a weak norm!!! (for weak uniqueness)
- $\|R(\hat{u})\|_{H^{-1}}$ & $\|hR(\hat{U})\|_{L_2}$ vs $\|R(\hat{u})\|_{L_2}$
- Weak uniqueness characterized by stability factor $S_\epsilon(\hat{\psi})$

G2 for NSE: Adaptive DNS/LES

NSE: $R(\hat{u}) = 0$

G2: $\hat{U} \in \hat{V}_h : (R(\hat{U}), \hat{v}) + SD_\delta(\hat{U}; \hat{v}) = 0 \quad \forall \hat{v} \in \hat{V}_h$

Functional output: $M(\hat{u}) = (\hat{u}, \hat{\psi})$ (drag, lift,...)

Dual NSE: $a(\hat{u}, \hat{w}; \hat{v}, \hat{\varphi}) = M(\hat{v}) \quad \forall \hat{v} \in \hat{V}_0$

$$a(\hat{u}, \hat{w}; \hat{v}, \hat{\varphi}) \equiv (\dot{v}, \varphi) + (u \cdot \nabla v, \varphi) + (v \cdot \nabla w, \varphi) - (\nabla \cdot \varphi, q) + (\nabla \cdot v, \theta) + (\nu \nabla v, \nabla \varphi)$$

Error identity (using duality and G2):

$$|M(\hat{u}) - M(\hat{U})| = |(R(\hat{U}), \hat{\varphi} - \hat{\Phi}) + SD_\delta(\hat{U}; \hat{\Phi})| \quad \forall \hat{\Phi} \in \hat{V}_h$$

G2 for NSE: Adaptive DNS/LES

$$|M(\hat{u}) - M(\hat{U})| \leq \sum_{K \in \mathcal{T}} \mathcal{E}_K = \sum_{K \in \mathcal{T}} \|hR(\hat{U})\| \|\hat{\varphi}_h\|_1 + |SD_\delta(\hat{U}; \hat{\varphi}_h)|$$

Galerkin discretization error + stabilization modeling error

Adaptive algorithm: From coarse mesh $\mathcal{T}^0$ do

(1) compute primal and dual problem on $\mathcal{T}^k$

(2) if $\sum_{K \in \mathcal{T}^k} \mathcal{E}_K^k < \text{TOL}$ then STOP, else

(3) refine elements $K \in \mathcal{T}^k$ with largest $\mathcal{E}_K^k \quad \to \quad \mathcal{T}^{k+1}$

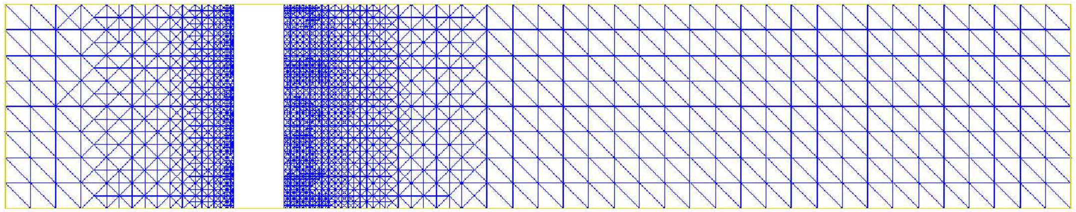
(4) set $k = k + 1$, then goto (1)

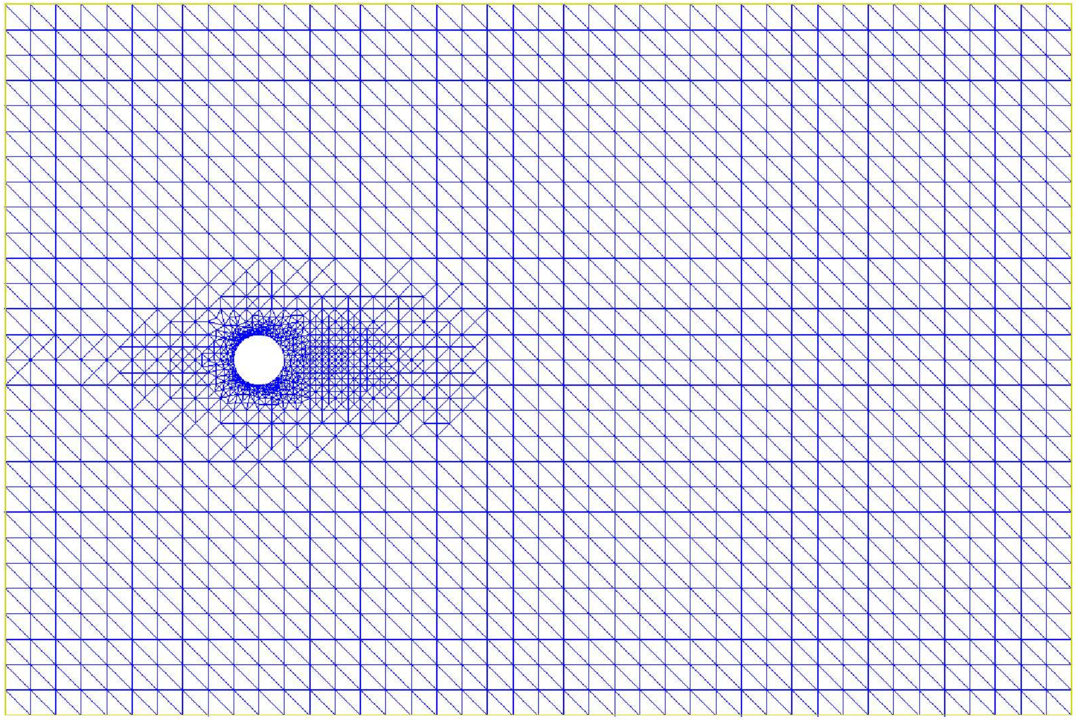
Bluff Body Benchmark Problems

Drag Coeff. c_D = normalized mean drag force

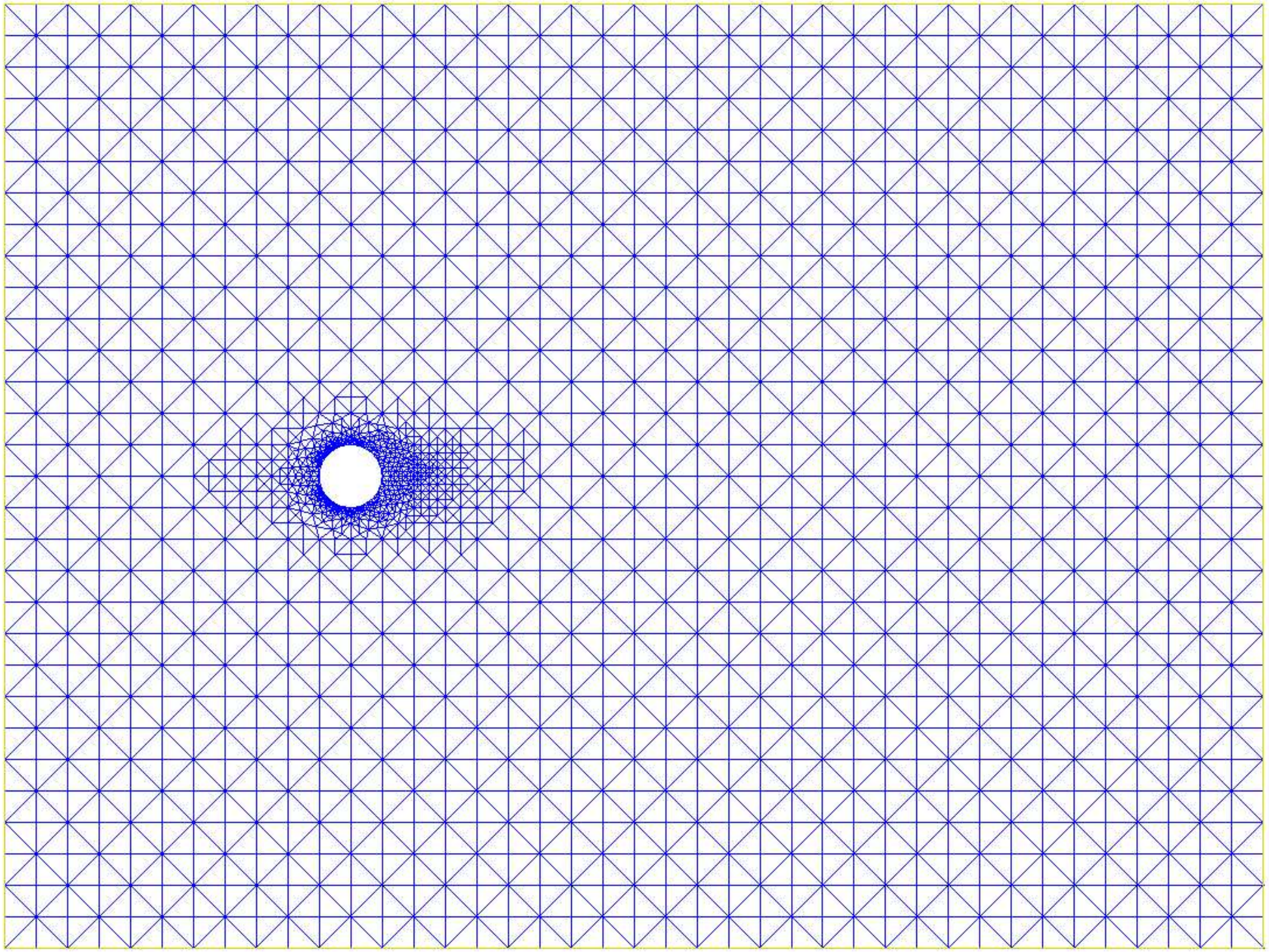
- *circular cylinder Re=3900*
- *dual solution*
- *sphere Re=10 000*
- *dual solution*
- *square cylinder Re=22 000*
- *dual solution*
- *cube Re=40 0000: xy* — *cube xz*
- *dual sol. xy* — *dual sol. xz*

Ref. Mesh wrt c_D: circular cylinder

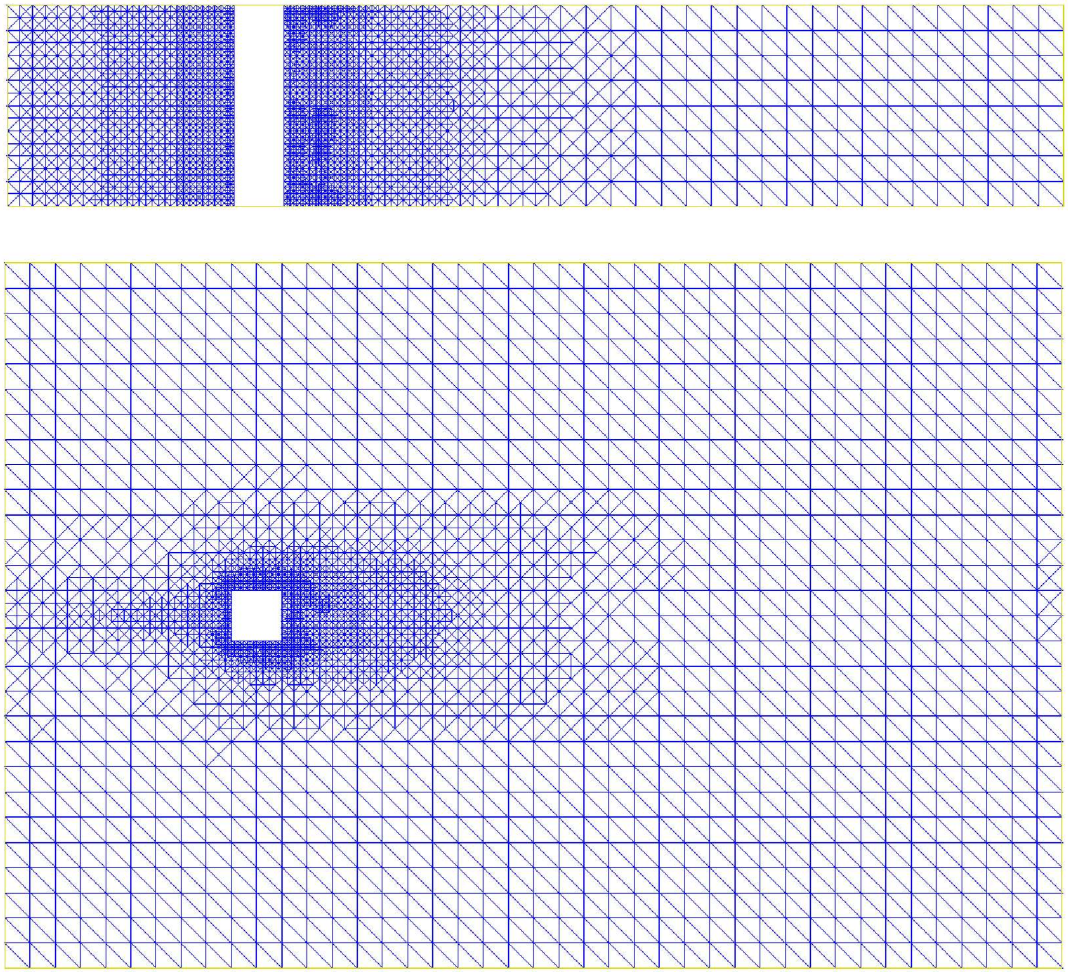




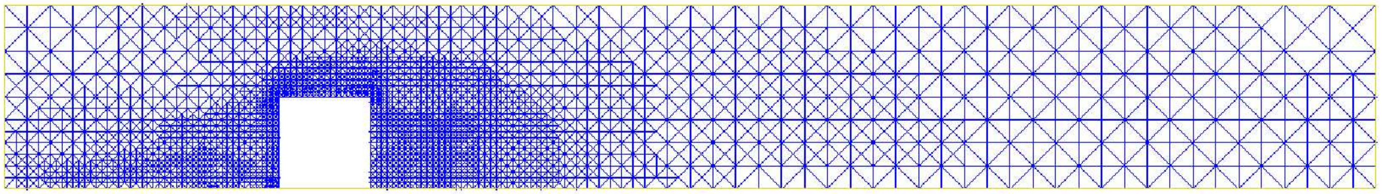
Ref. Mesh wrt c_D: sphere

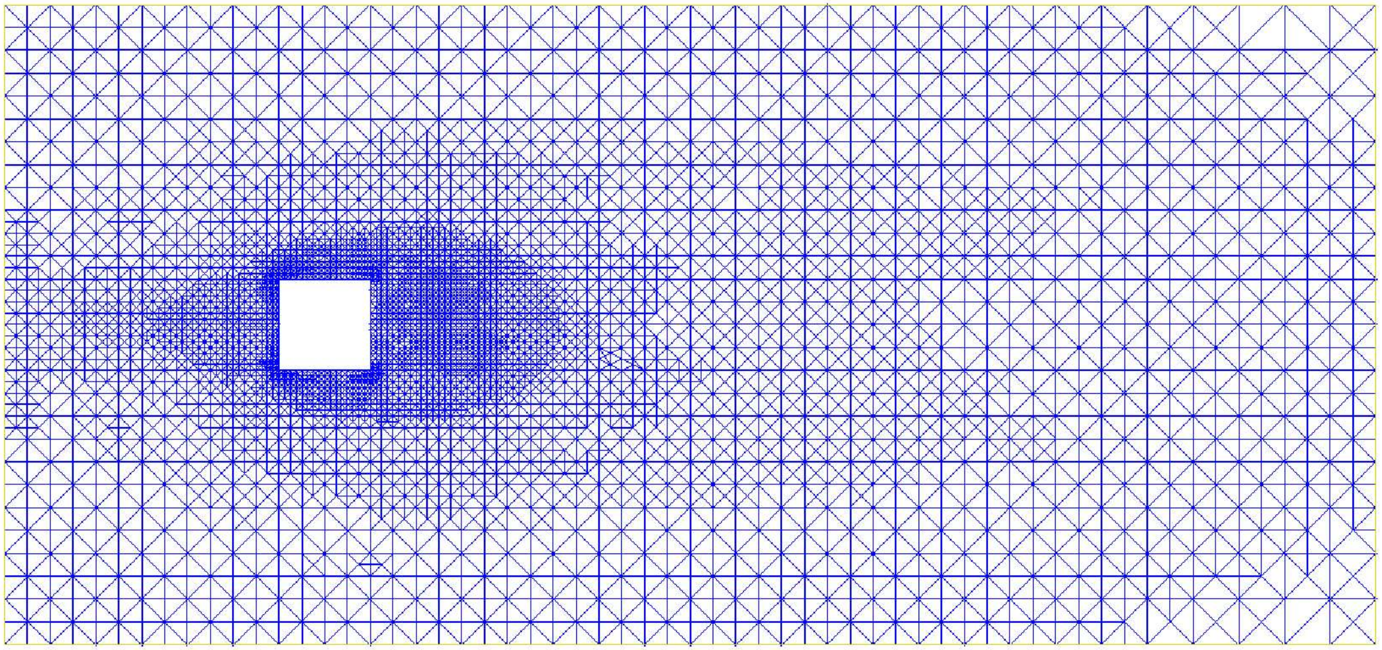


Ref. Mesh wrt c_D: square cylinder

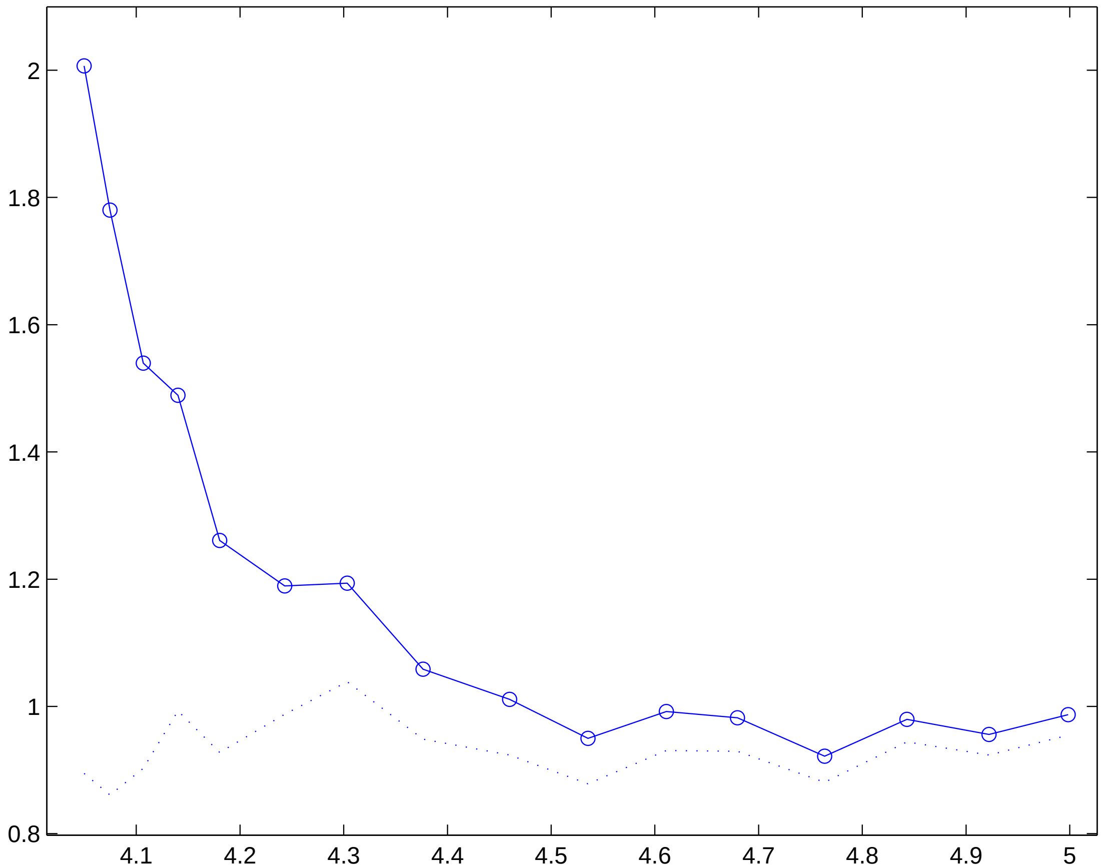


Ref. Mesh wrt c_D: surf mount cube

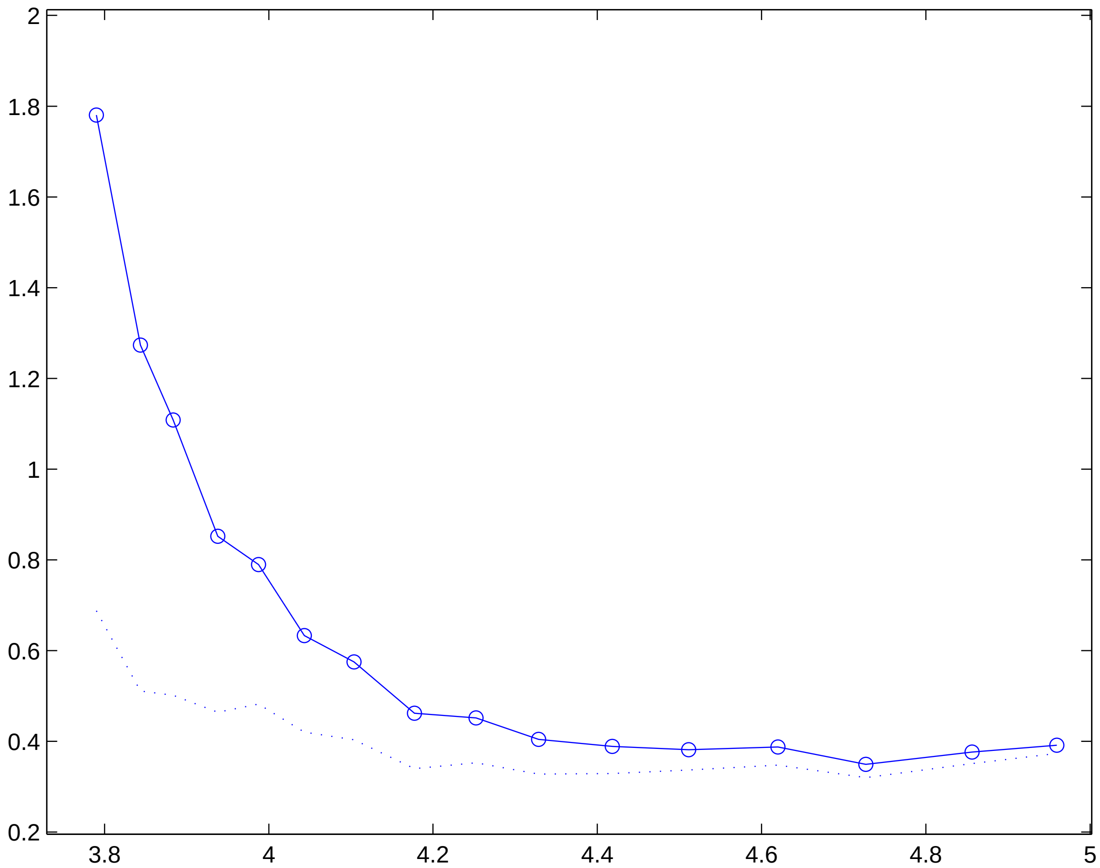




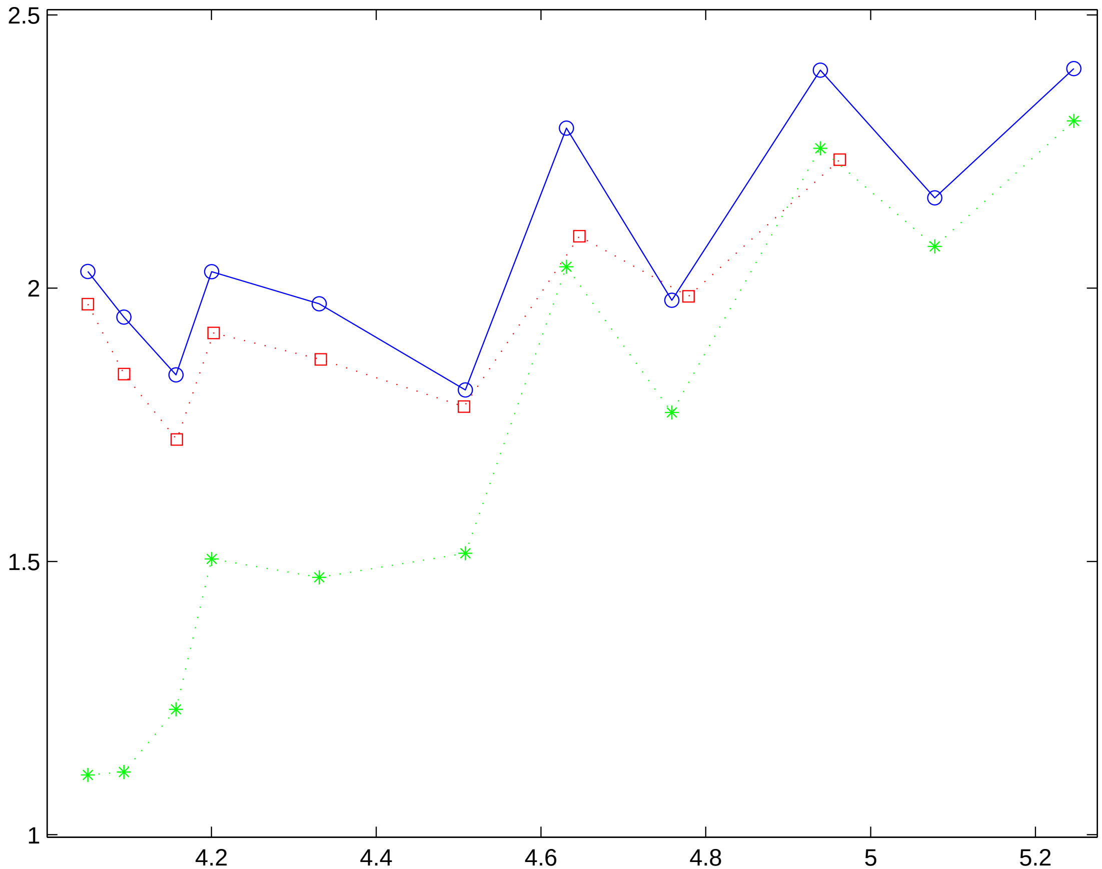
Circular cylinder: $c_D \approx 1.0$



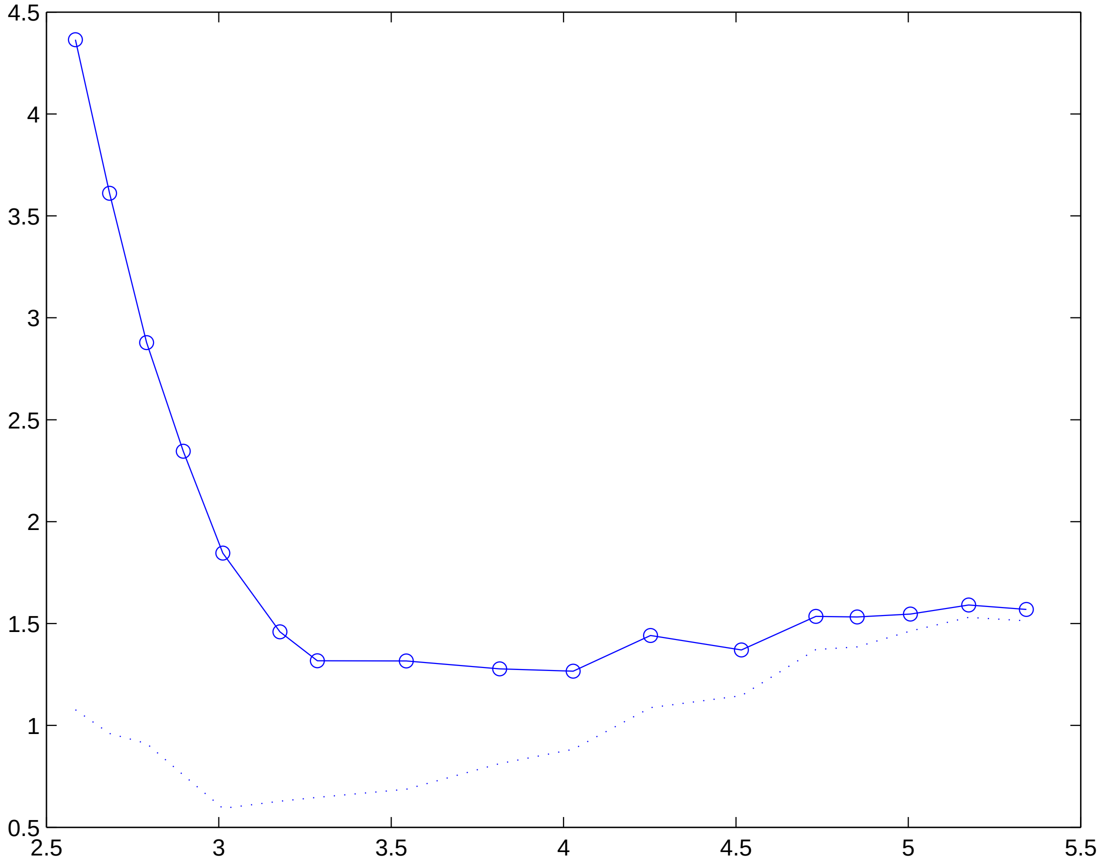
Sphere: $c_D \approx 0.4$



Square cylinder: $c_D \approx 2.2$



Surface mounted cube: $c_D \approx 1.5$

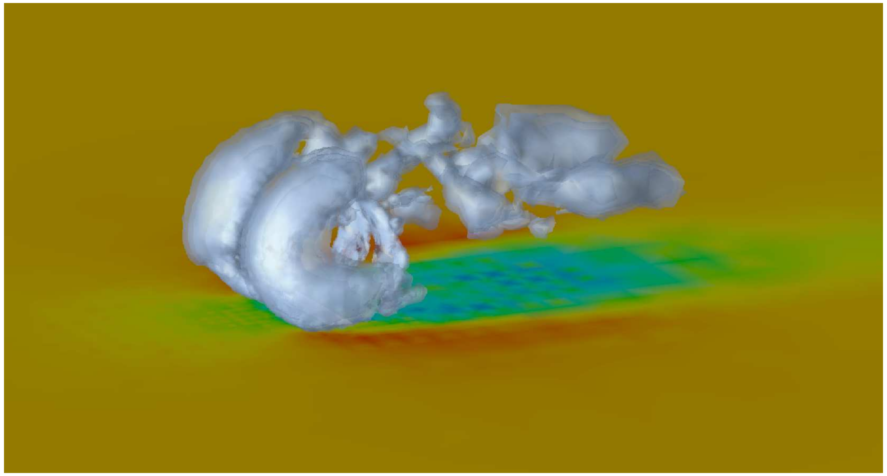


Summary Benchmark Problems

- G2 for NSE: No filtering. No Reynolds stresses. G2 automatic “turbulence model”.
- Adaptive algorithm captures separation points, and “correct” (finite limit) dissipation in the turbulent wake.
- Mean value output (drag, lift, frequencies, separation points, pressure coeff,...) computable up to a tolerance corresponding to experimental accuracy ($\approx$ 1-5%).
- About 10-100 times less mesh points needed to compute drag than in non-adaptive LES.
- All computations on a standard PC (2 GHz, 0.5Gb)

Ex: Rotating Cylinder $Re = 10^4$

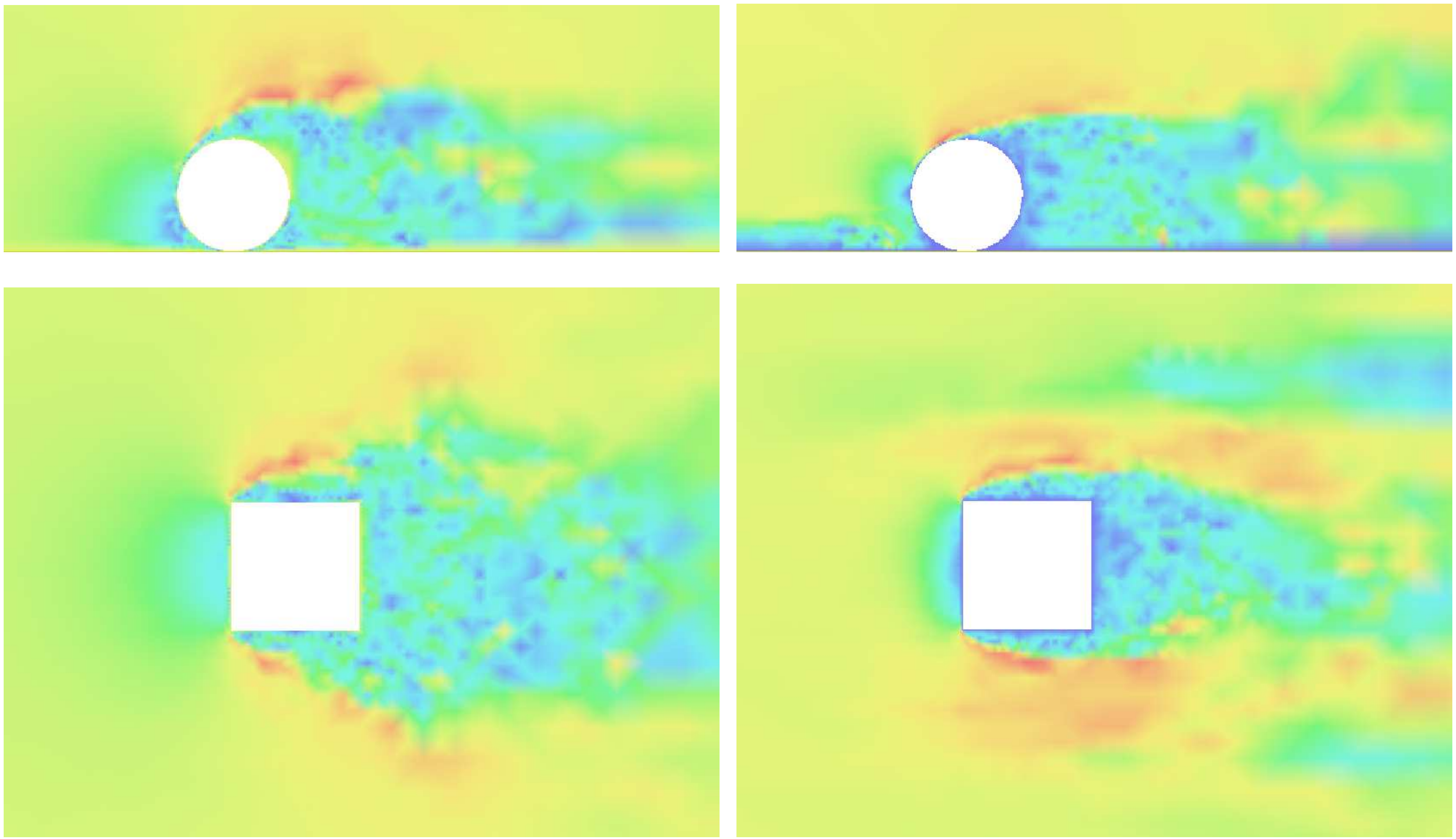
Rotating cylinder, ground moving at free stream speed



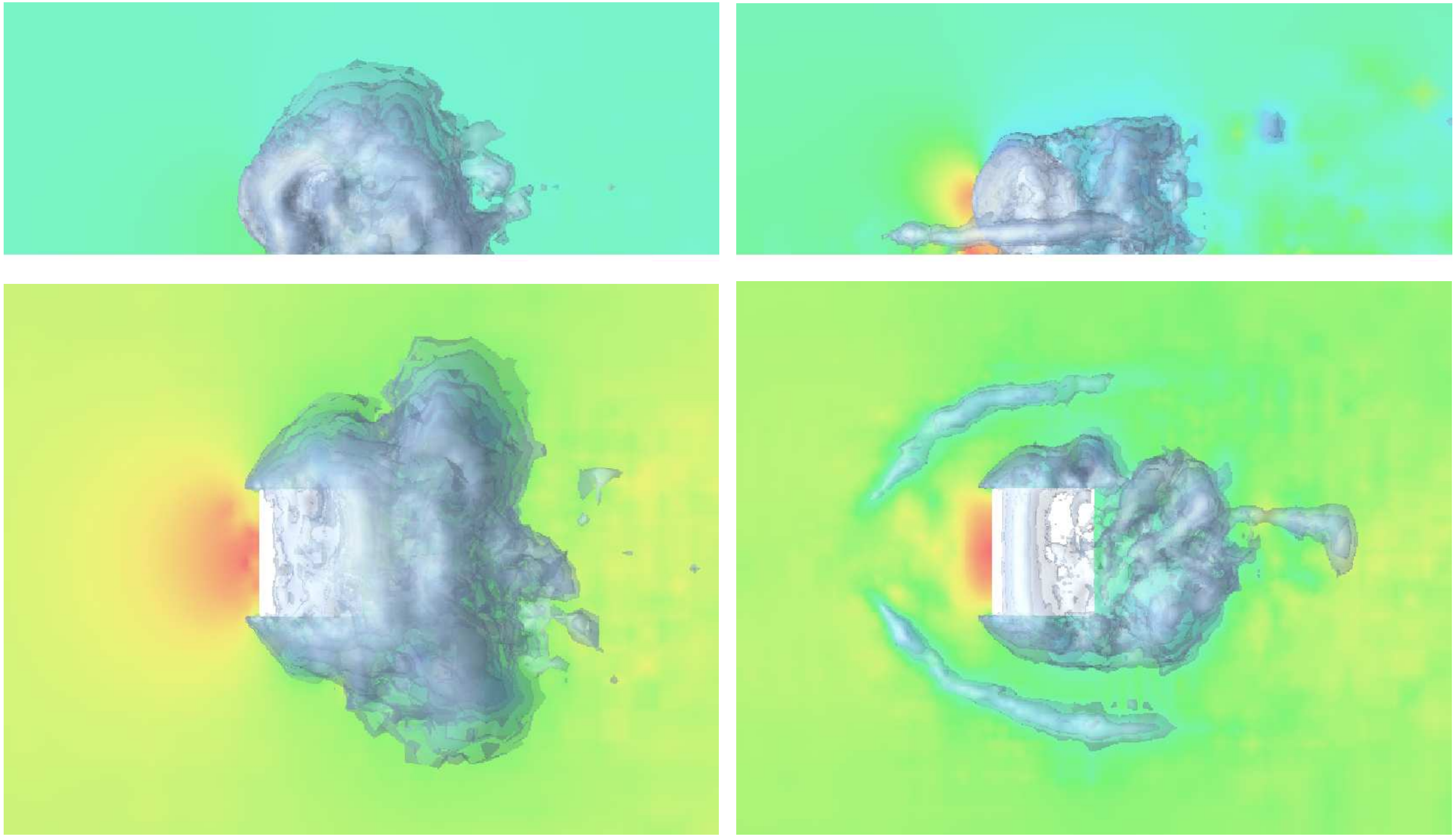
Model of rotating cyl on ground: moving coordinate frame
(Wheels of F1 racing car, airplane at take-off or landing,...)
Compare with stationary wheel (simple wind tunnel testing)

velocity xz *velocity xy* *transversal velocities*

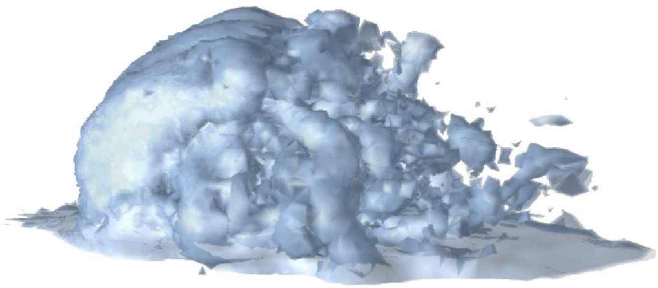
G2: rot vs stat: velocity

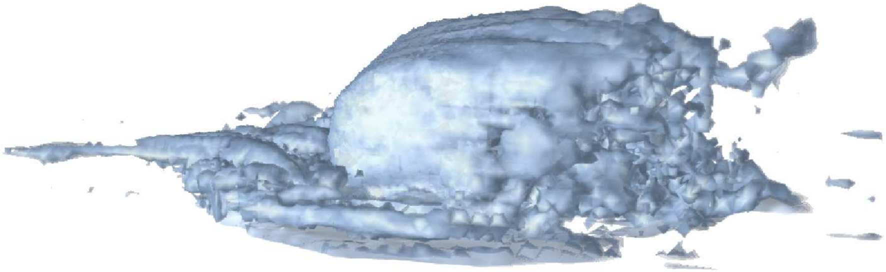


G2: rot vs stat: pressure

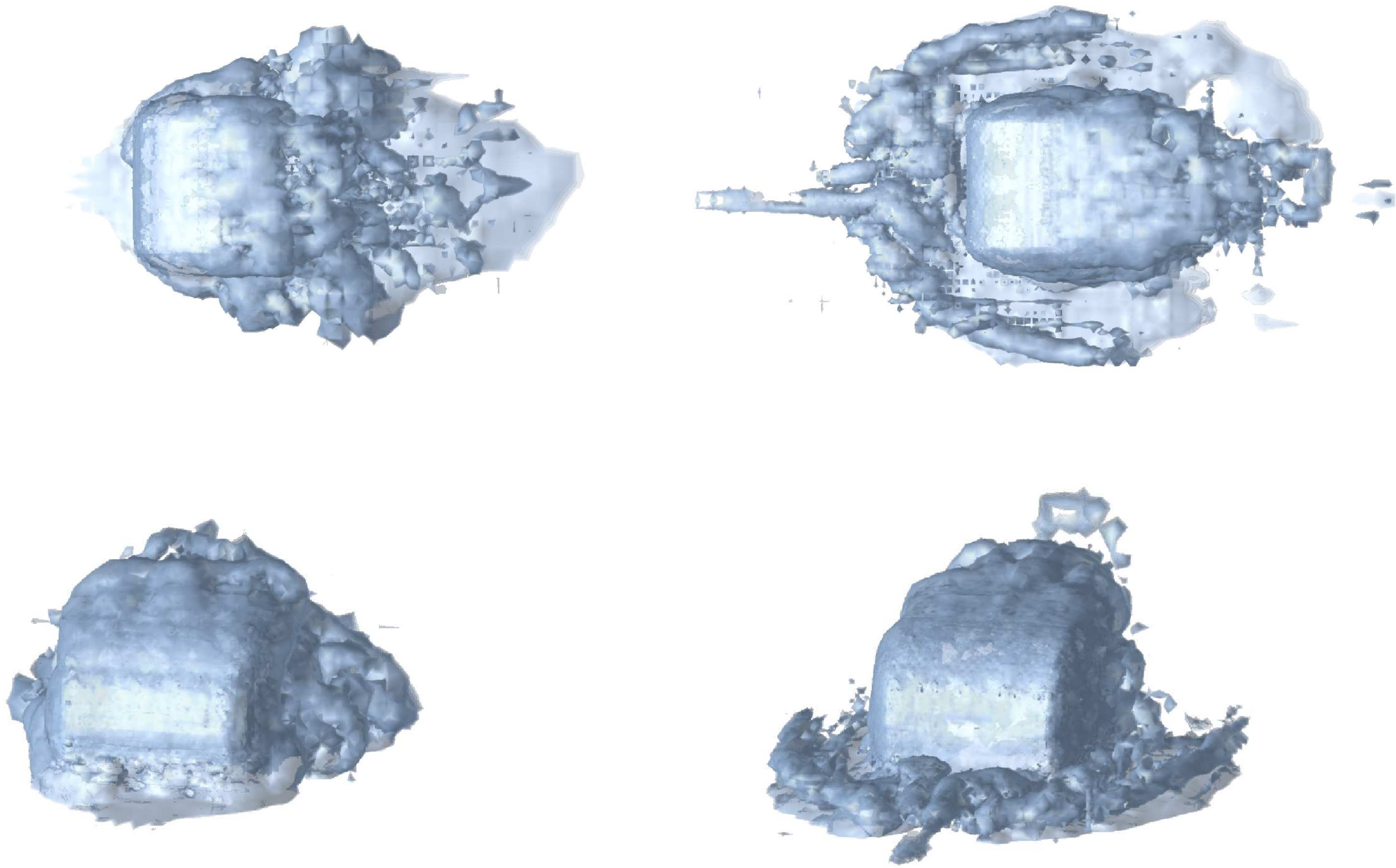


G2: rot vs stat: vorticity

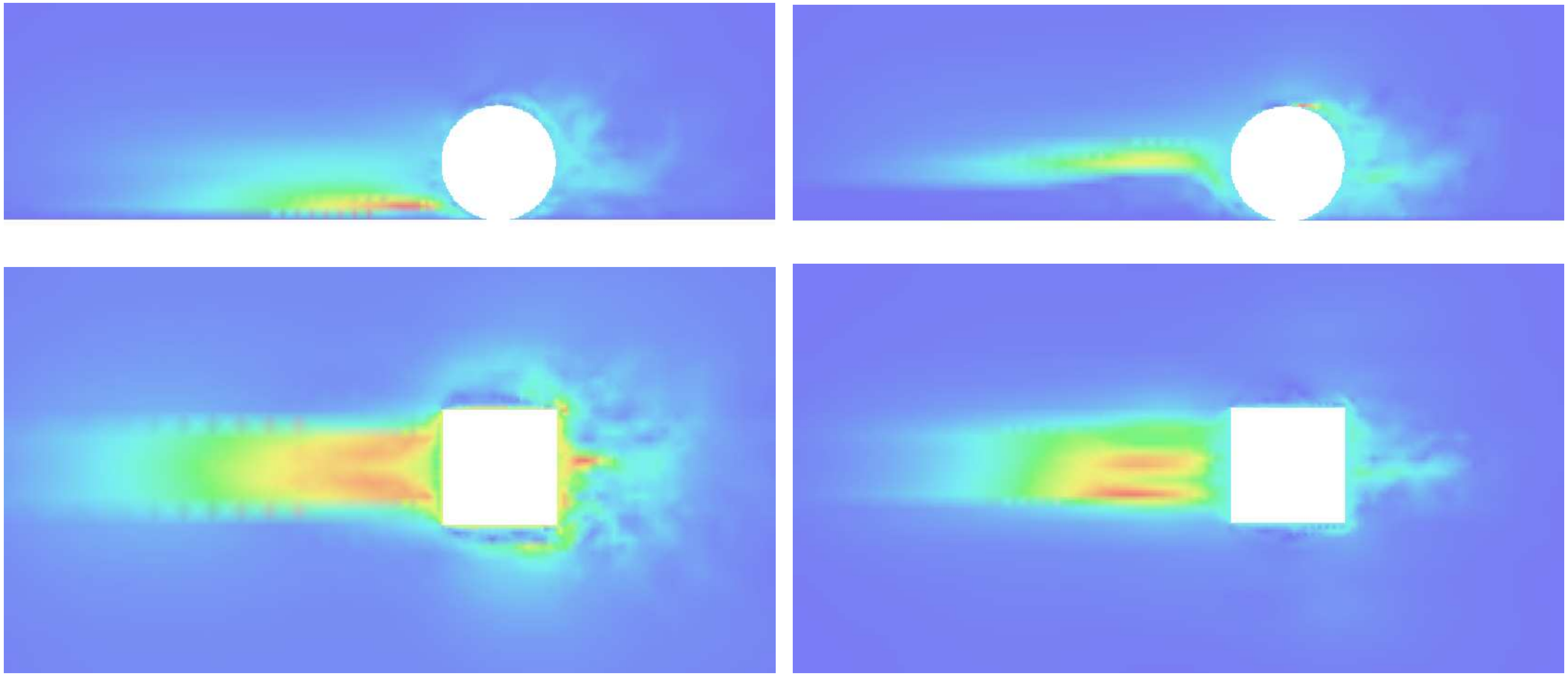




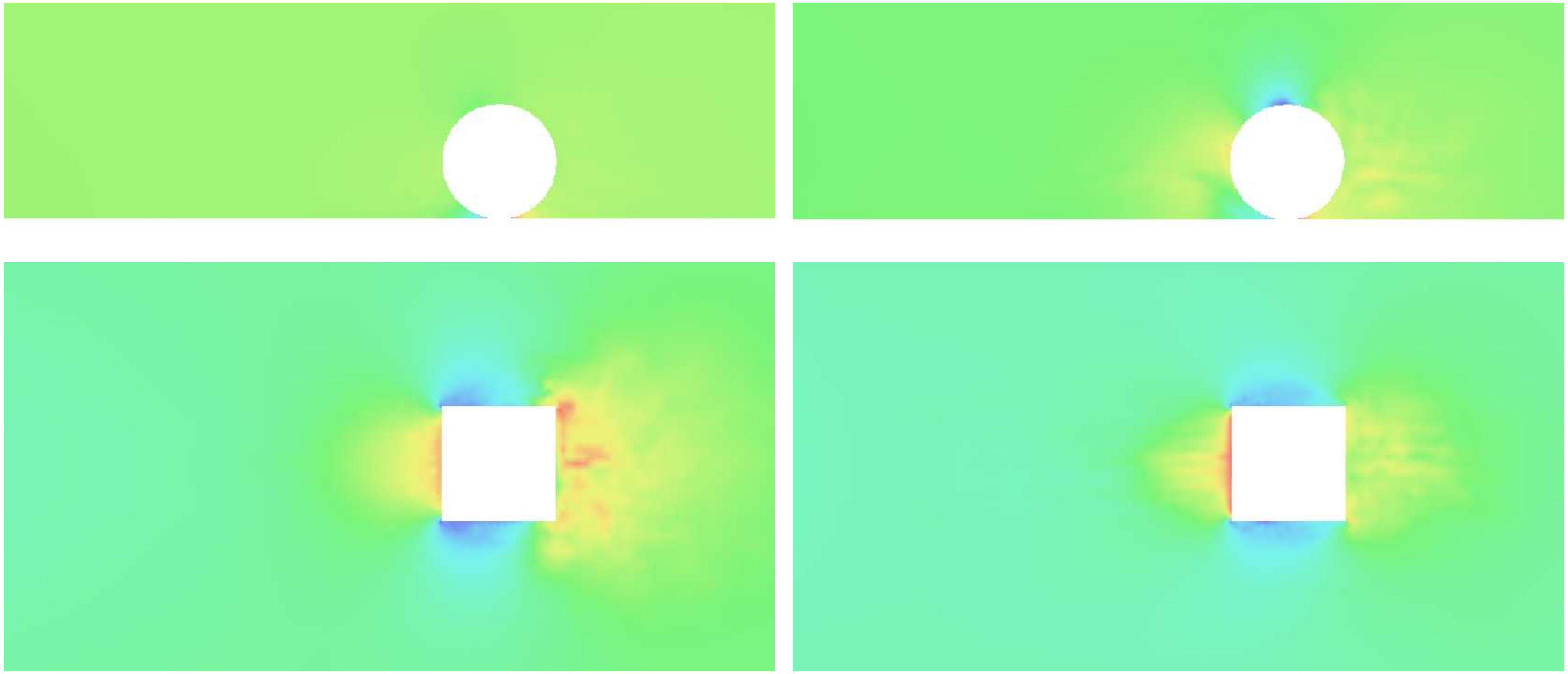
G2: rot vs stat: vorticity



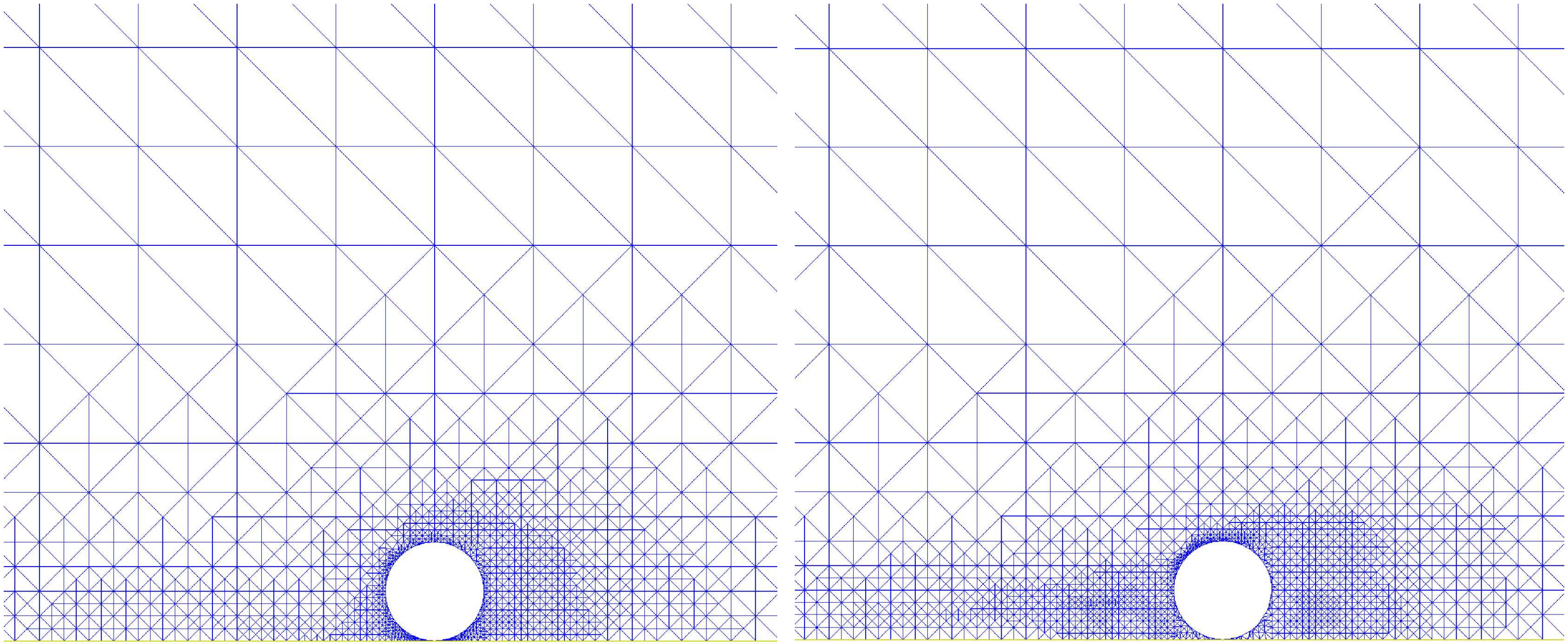
G2: rot vs stat: dual velocity



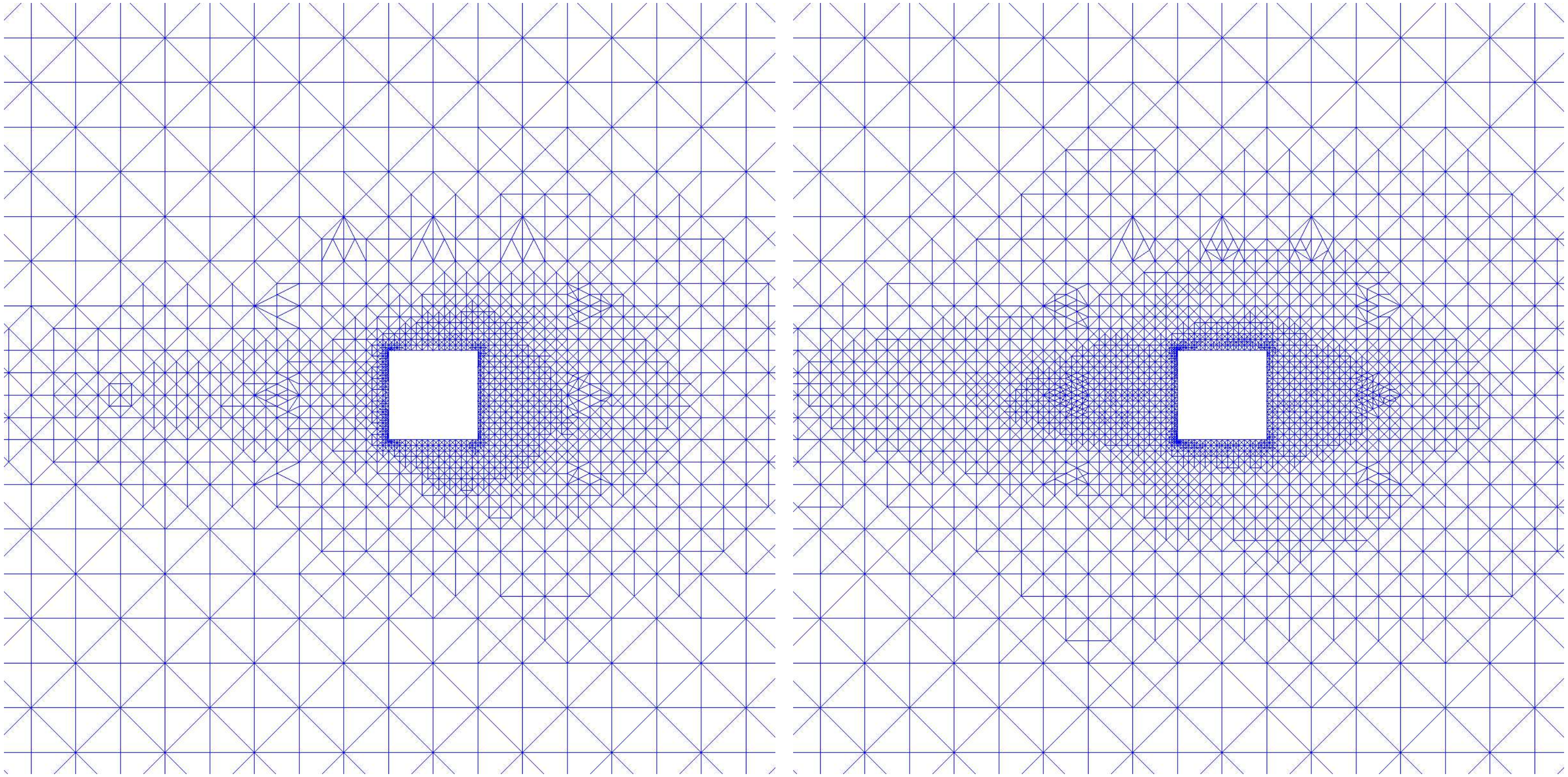
G2: rot vs stat: dual pressure



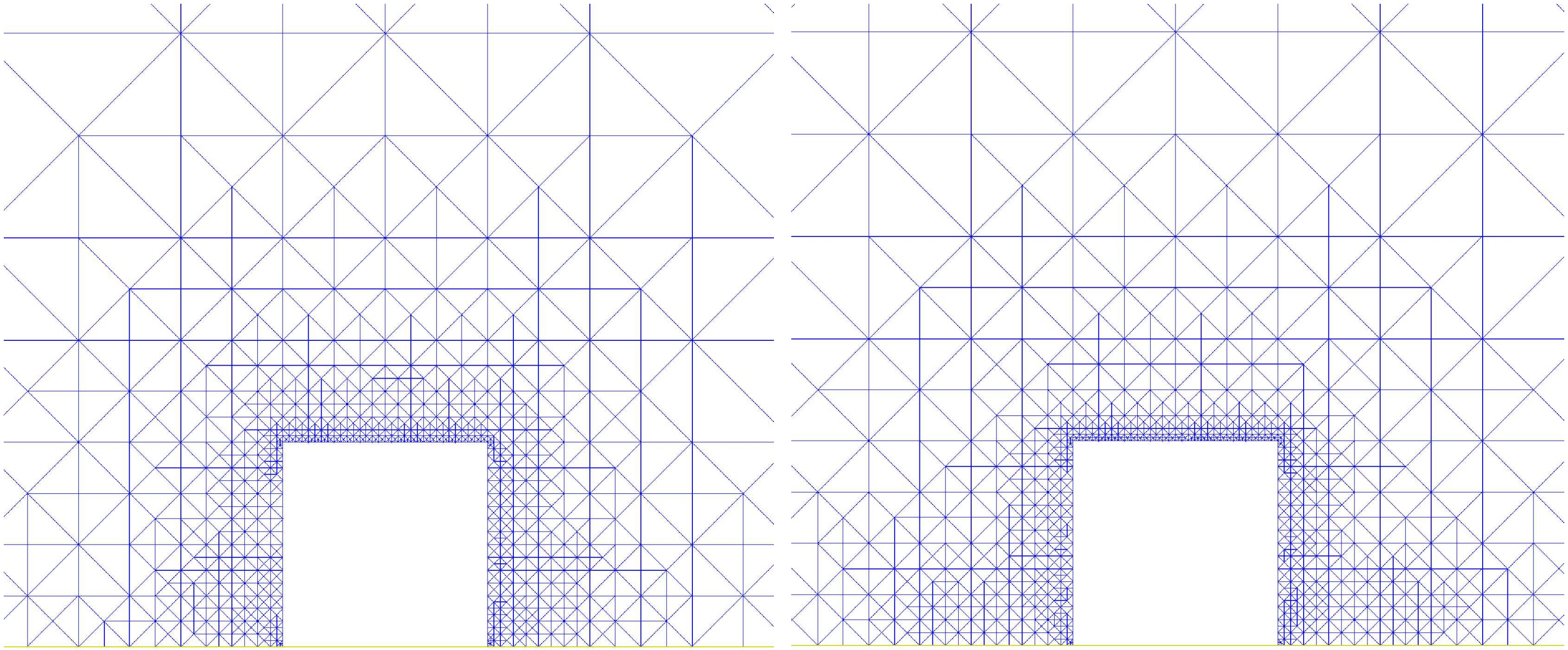
G2: rot vs stat: mesh



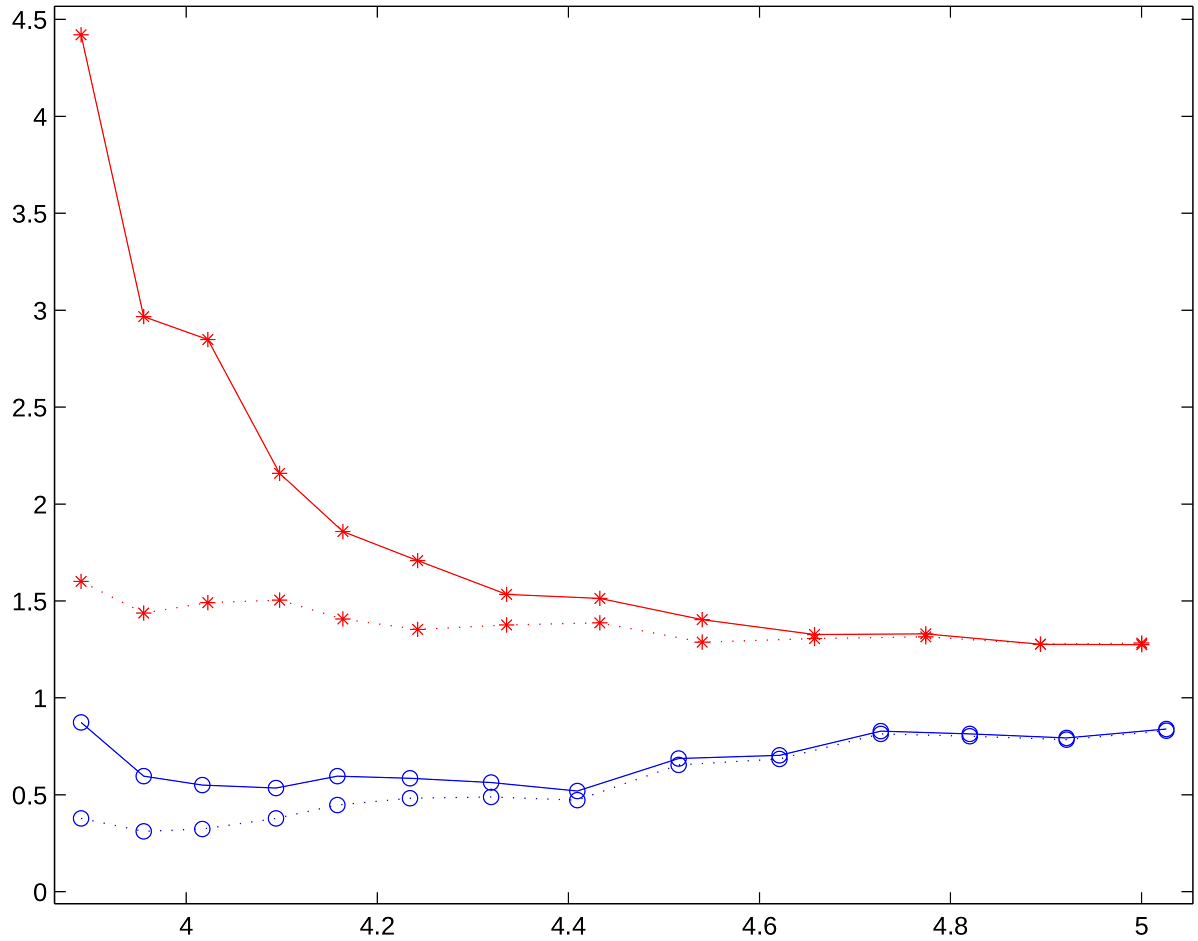
G2: rot vs stat: mesh



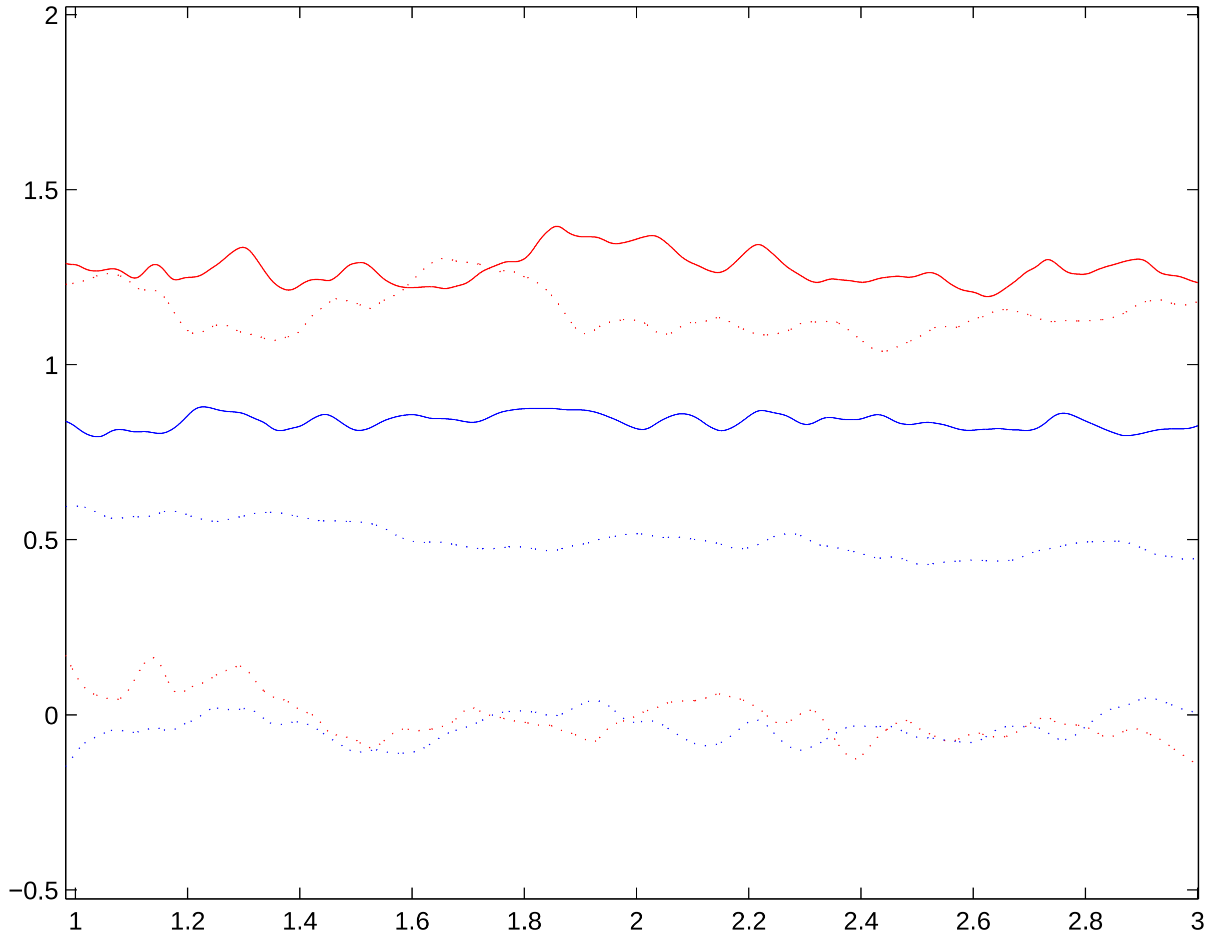
G2: rot vs stat: mesh



G2: rot vs stat: drag c_D: 1.3 vs 0.8



G2: rot vs stat: forces

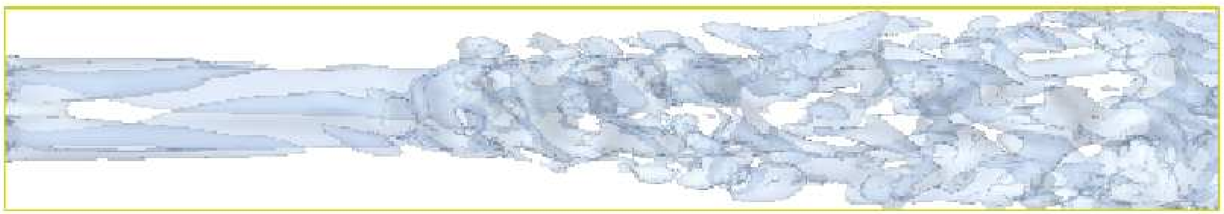


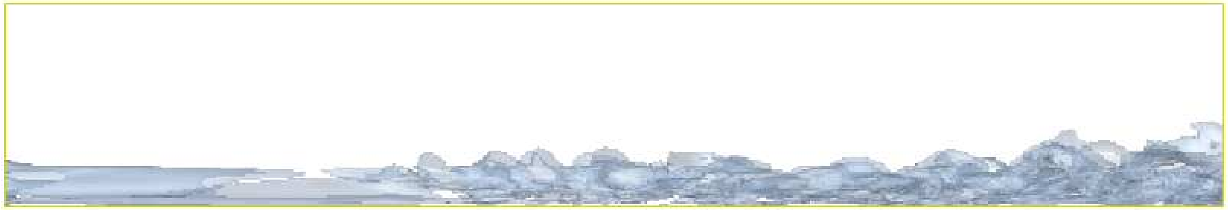
Turbulent boundary layers

So far: No slip boundary conditions seems ok for modeling laminar boundary layers (separation and skin friction)

For high Re boundary layer undergoes transition

Extremely expensive to resolve turbulent boundary layer:
$\Rightarrow$ We need wall-modeling



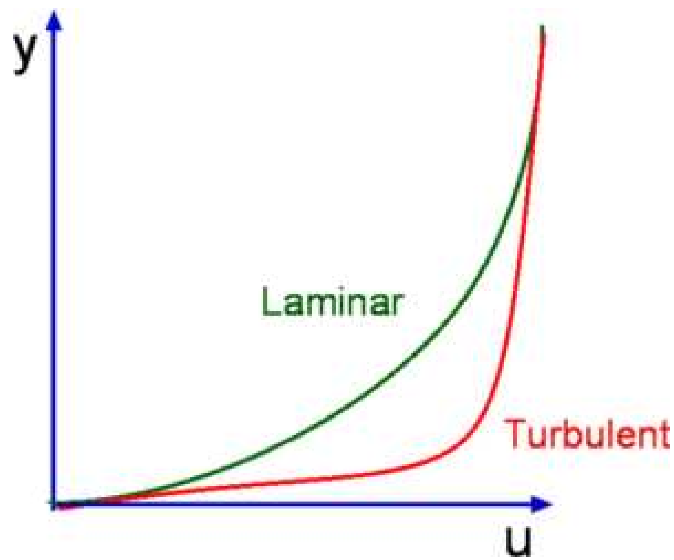


Turbulent boundary layers

Separation given by: $\dot{u} + u \cdot \nabla u - \nu \Delta u = -\nabla p$

Retarding flow near the boundary: $\nabla p > 0$

Turbulent boundary layer $\Rightarrow$ higher momentum near the boundary $\Rightarrow$ delayed separation



Friction boundary condition

Slip with friction boundary condition [Maxwell, Navier,....]

Friction coefficient β; $\beta = 0$: slip b.c., $\beta = \infty$: no slip b.c.

[LES + Boundary Layer theory: Layton, John, Illiescu,....]

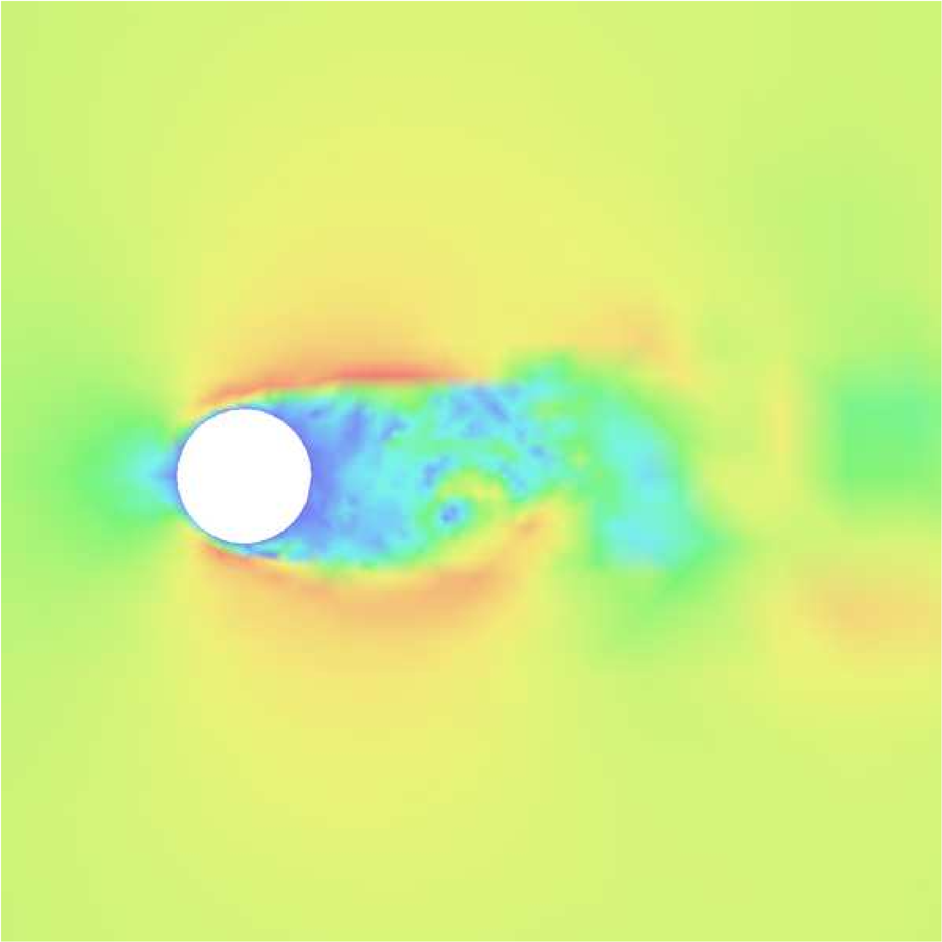
Simple wall model: $\beta \approx c_f$ (skin friction $c_f \sim Re^{-0.2}$)

$\beta = \beta(Re, h)$; $\lim_{h \to 0} \beta = \infty$, $\lim_{Re \to \infty} \beta = 0$ ($\lim_{Re \to 0} \beta = \infty$)

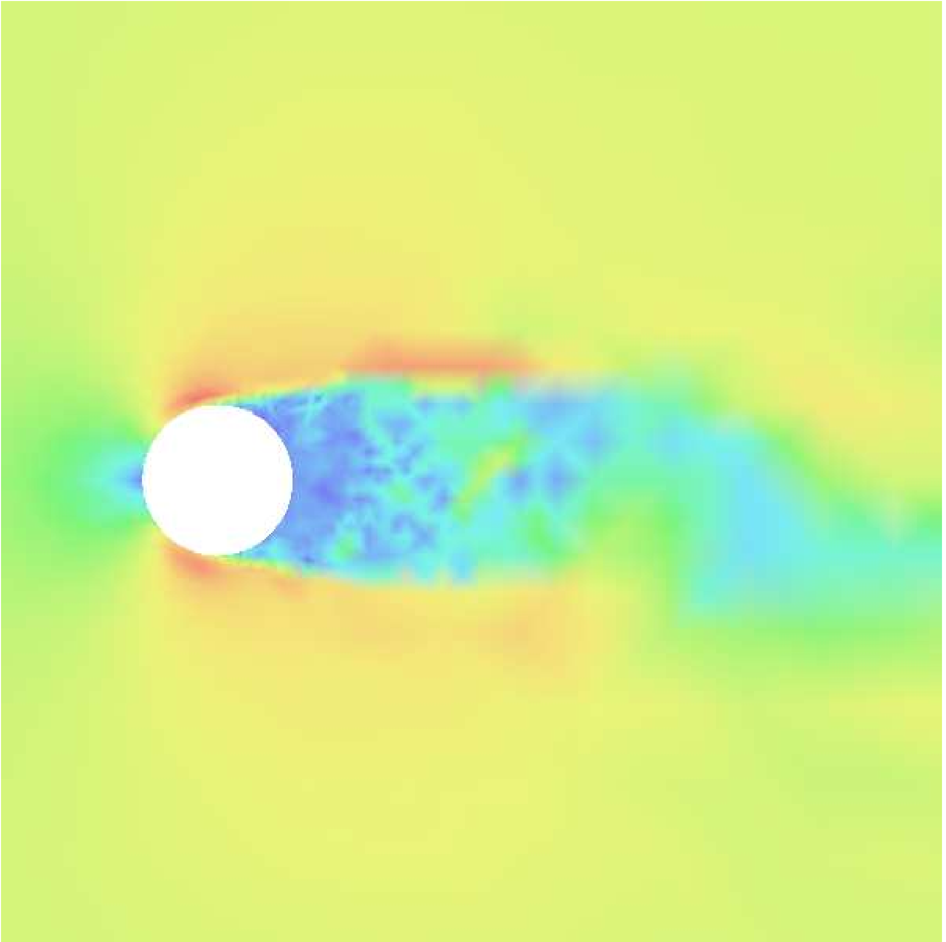
$$\frac{1}{2}||U(t)||^2 + \sum_{i=1}^{2} ||\sqrt{\beta} u \cdot \tau_i||^2_{\Gamma \times I} + ||\sqrt{h} R(\hat{U})||^2_Q = \frac{1}{2}||U(0)||^2$$

(with ν small)

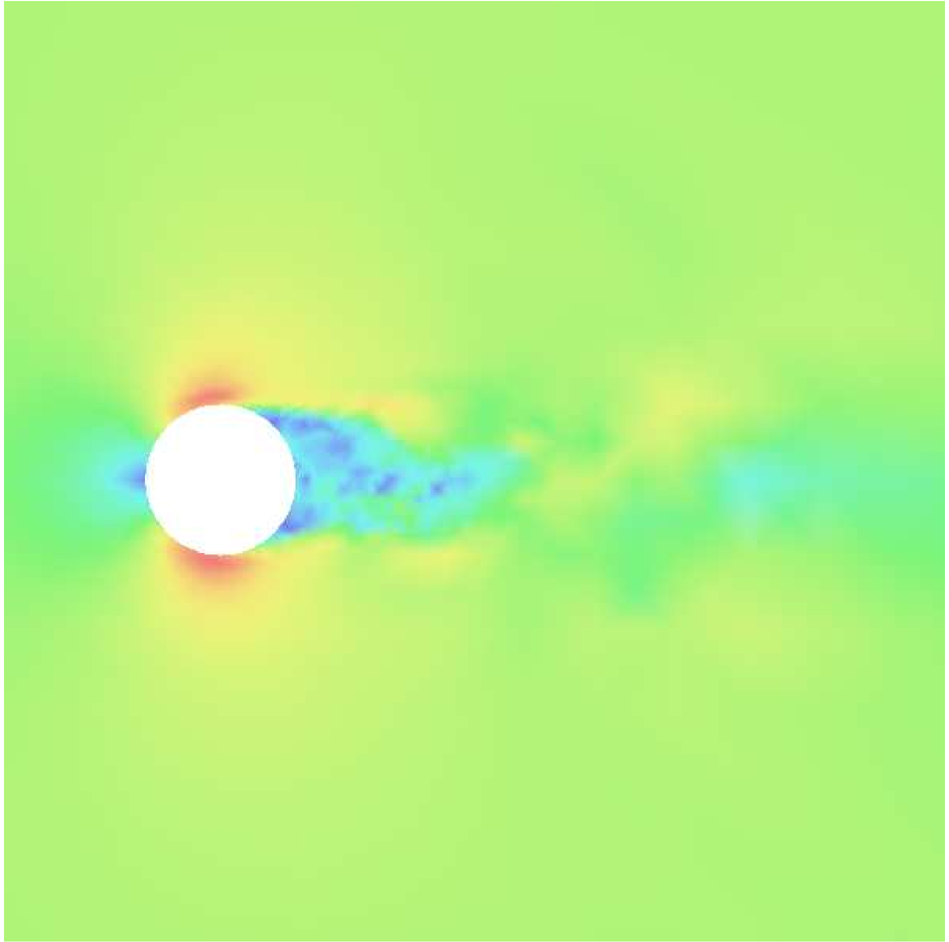
drag crisis; $\beta = 1$: $c_D \approx 1.0$



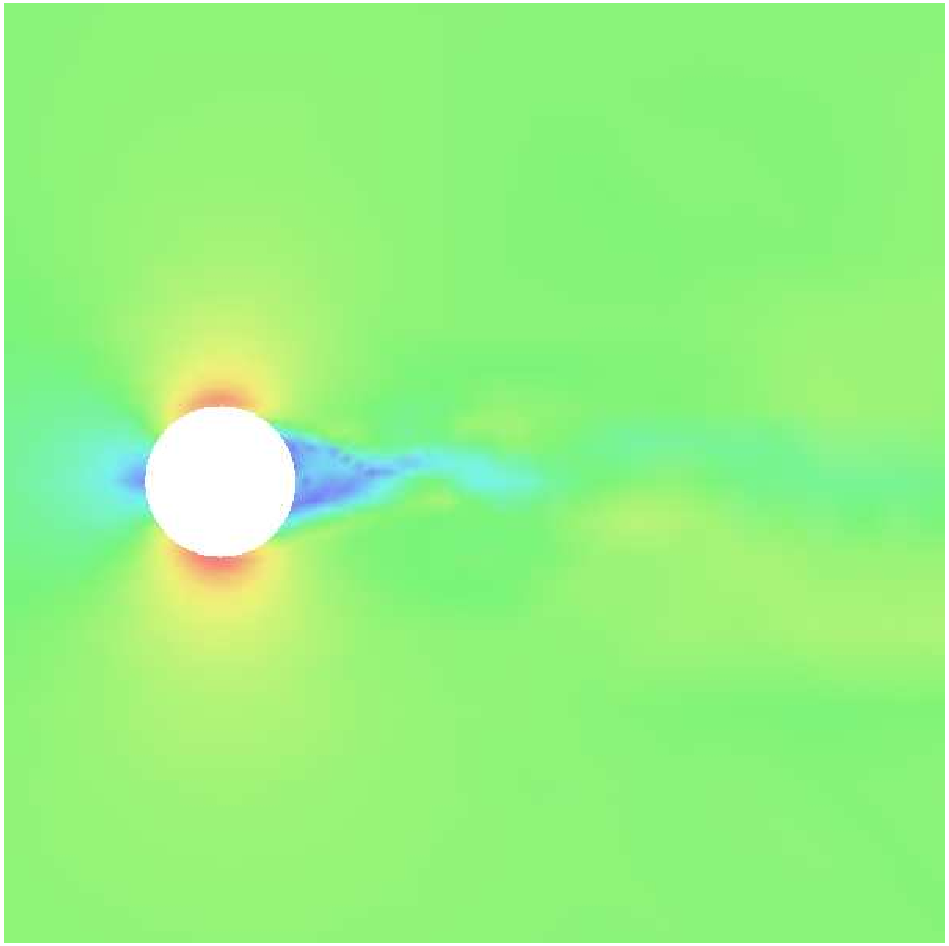
drag crisis; $\beta = 2 \times 10^{-2}$: $c_D \approx 0.7$



drag crisis: $\beta = 1 \times 10^{-2}$; $c_D \approx 0.5$



drag crisis; $\beta = 5 \times 10^{-3}$: $c_D \approx 0.45$

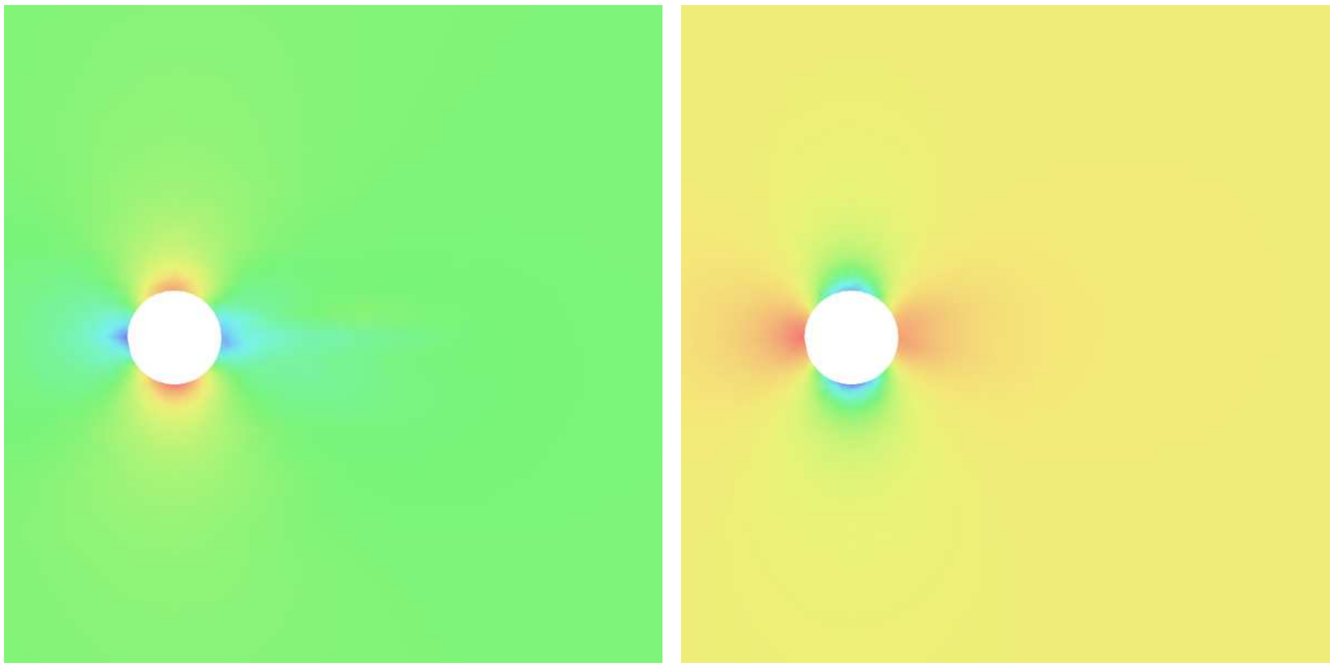


EG2 and Turbulent Euler solutions

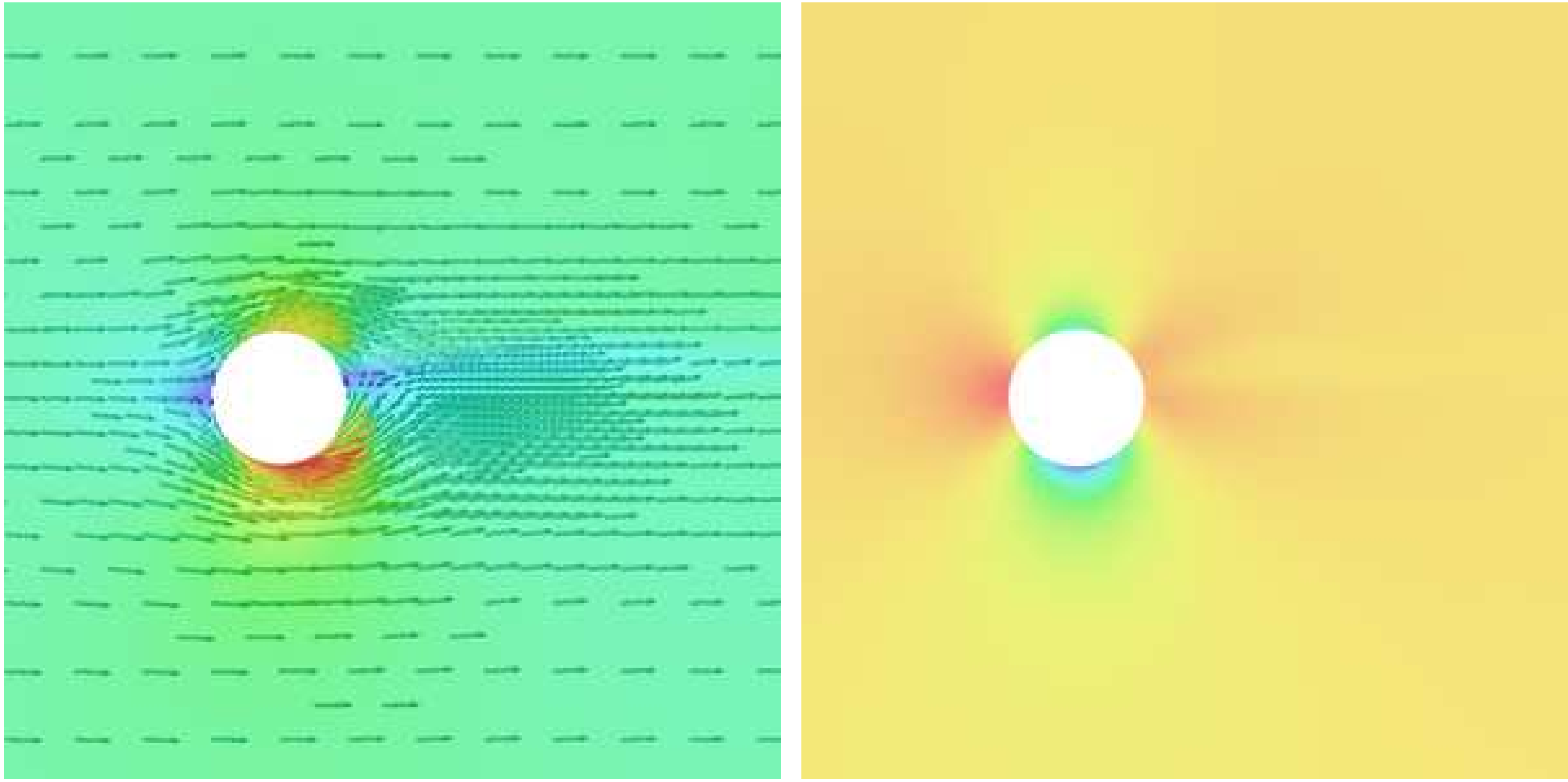
$\beta \to 0$; $Re \to \infty$ ($\nu \to 0$) $\Rightarrow$ Euler/G2 + slip b.c. (EG2)

EG2: no empirical parameters; only h (very general...)

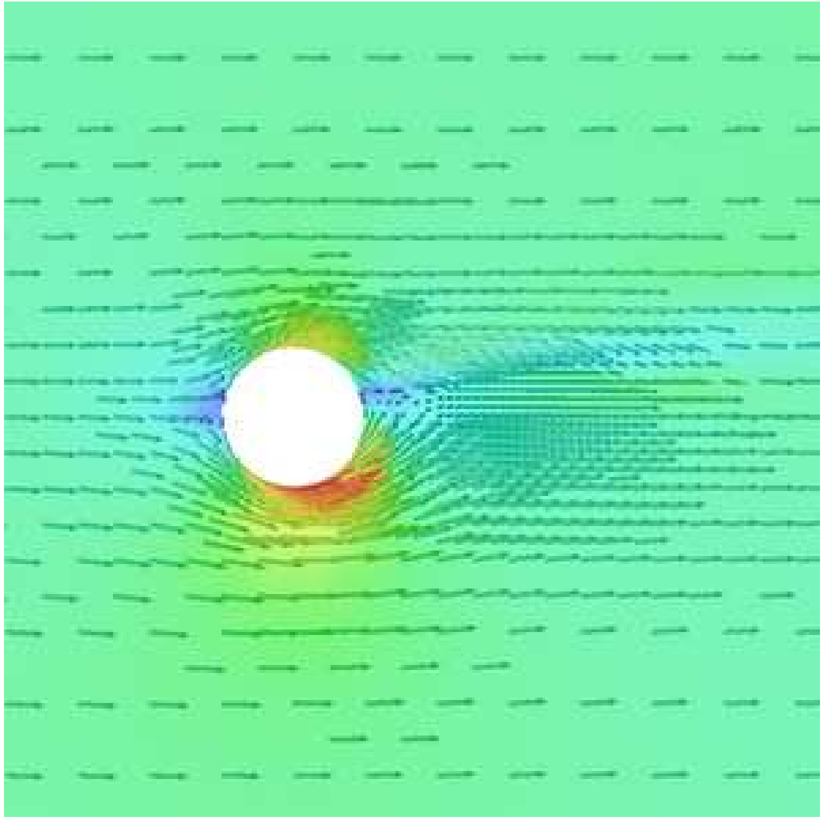
What happens in the limit? The potential solution ($c_D = 0$)?

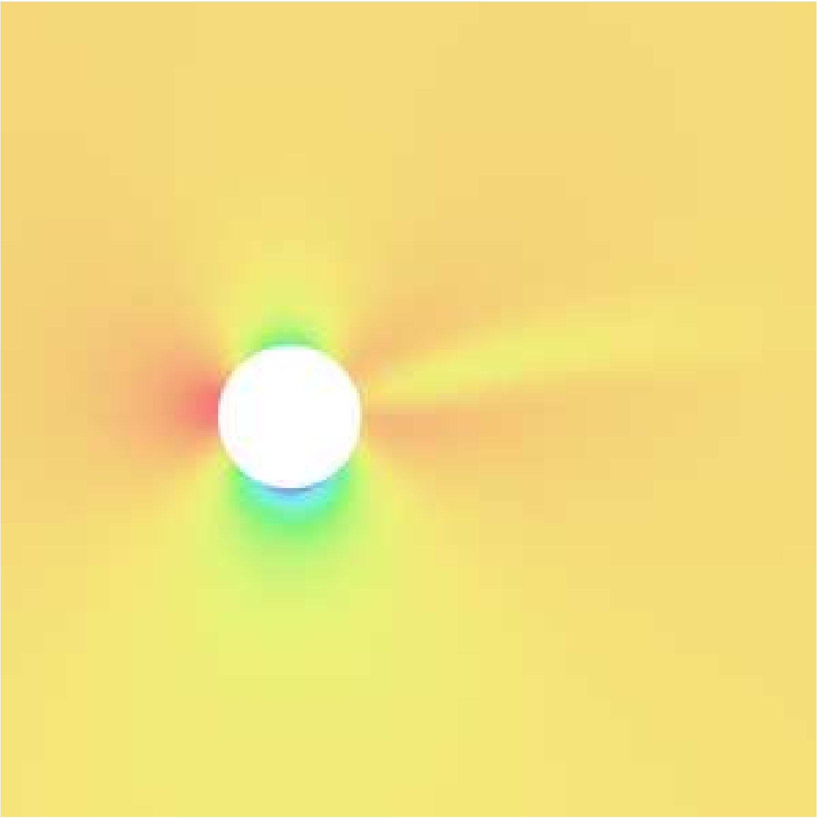


Velocity & pressure: t=1.0: $c_D = 0.22$

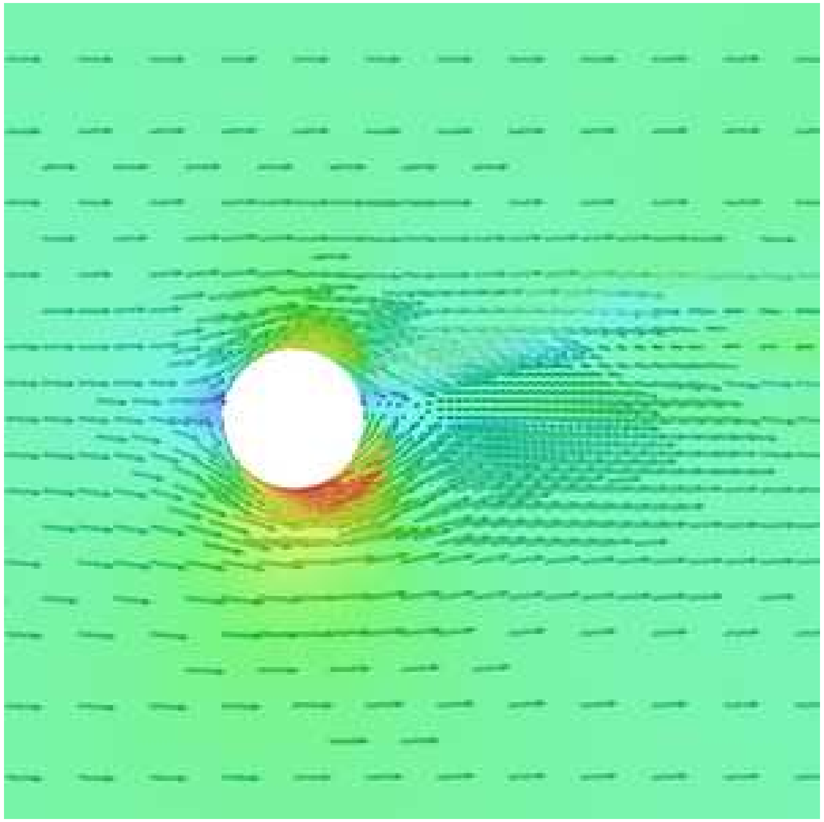
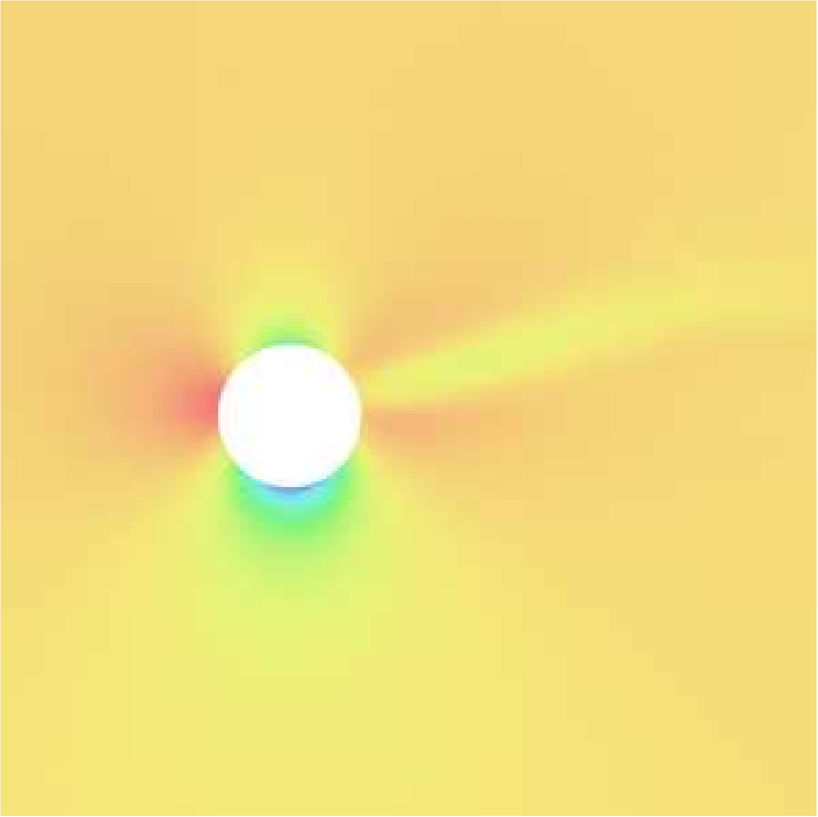


Velocity & pressure: t=1.25: $c_D = 0.25$

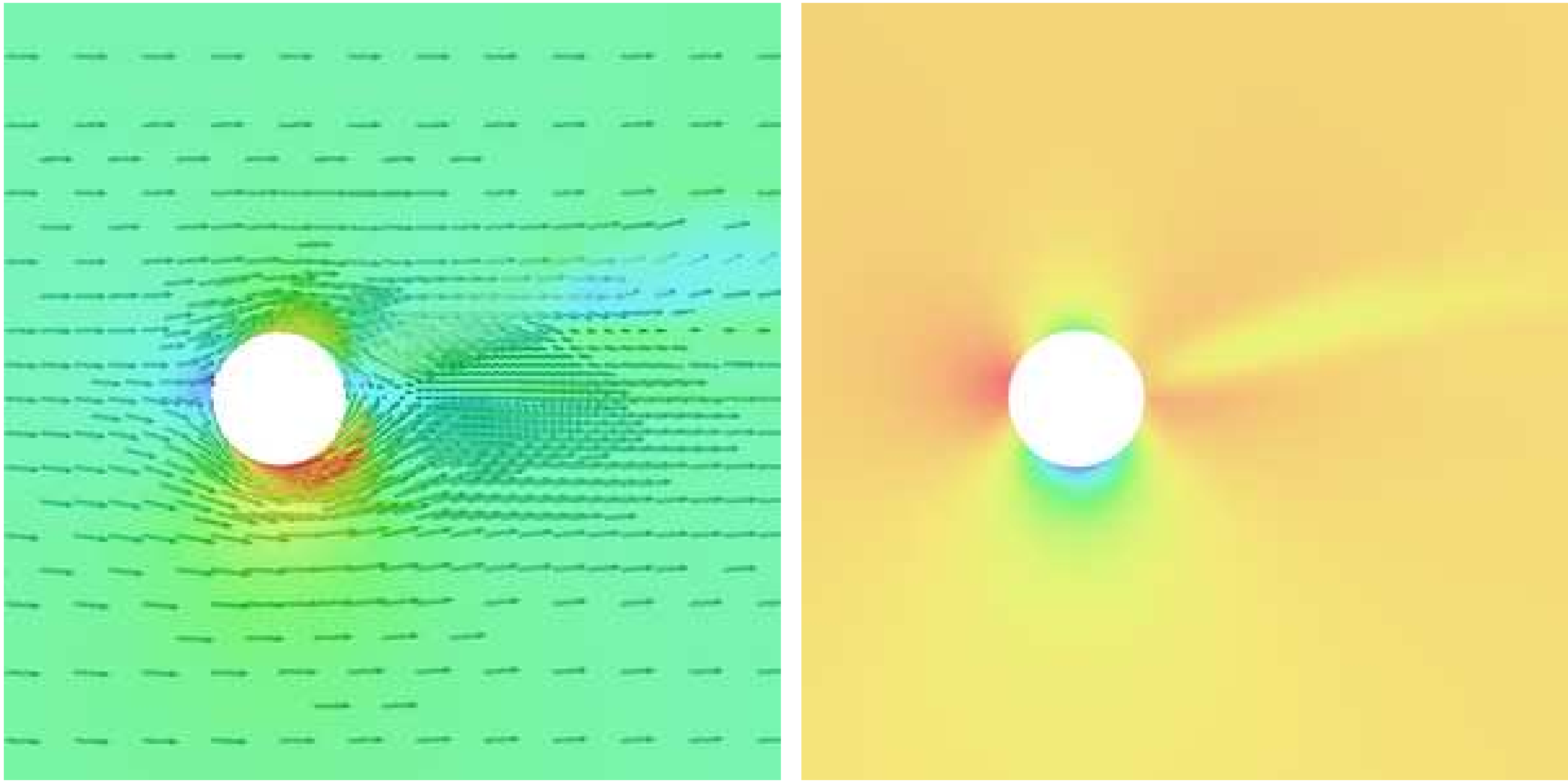




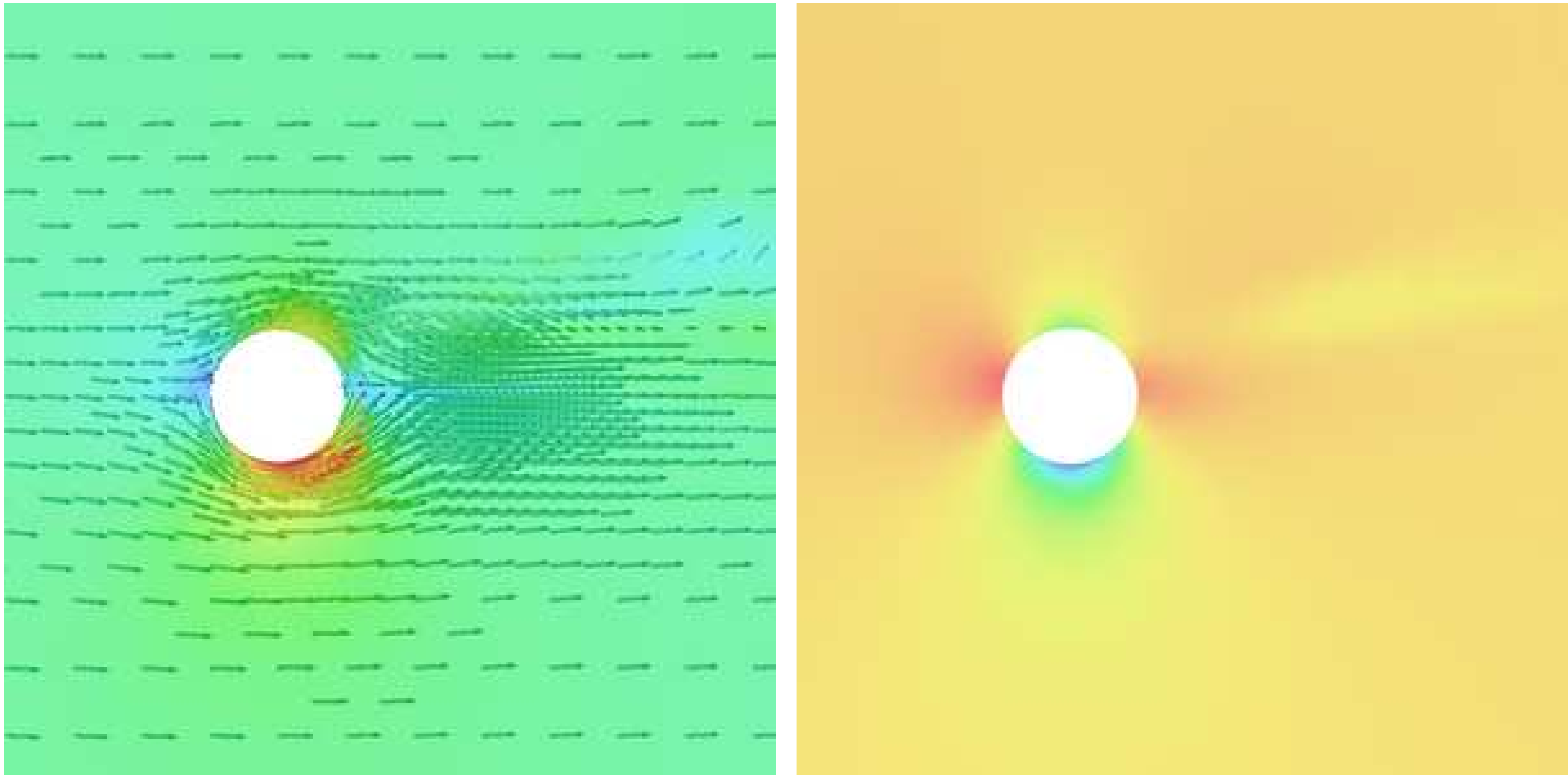
Velocity & pressure: t=1.5: $c_D = 0.28$

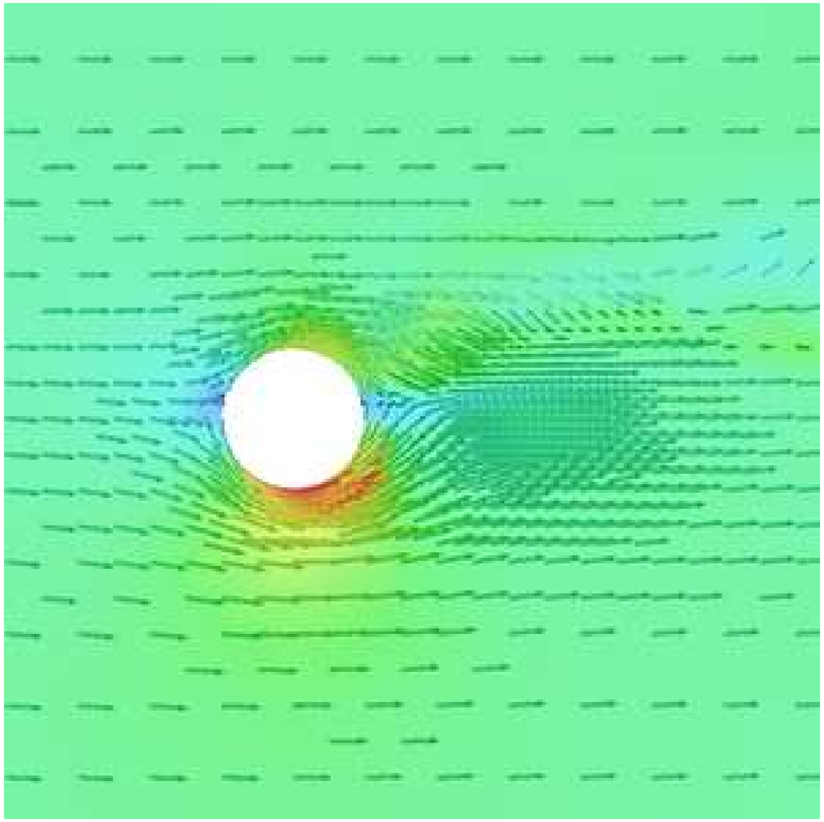
Velocity & pressure: t=1.75: $c_D = 0.36$

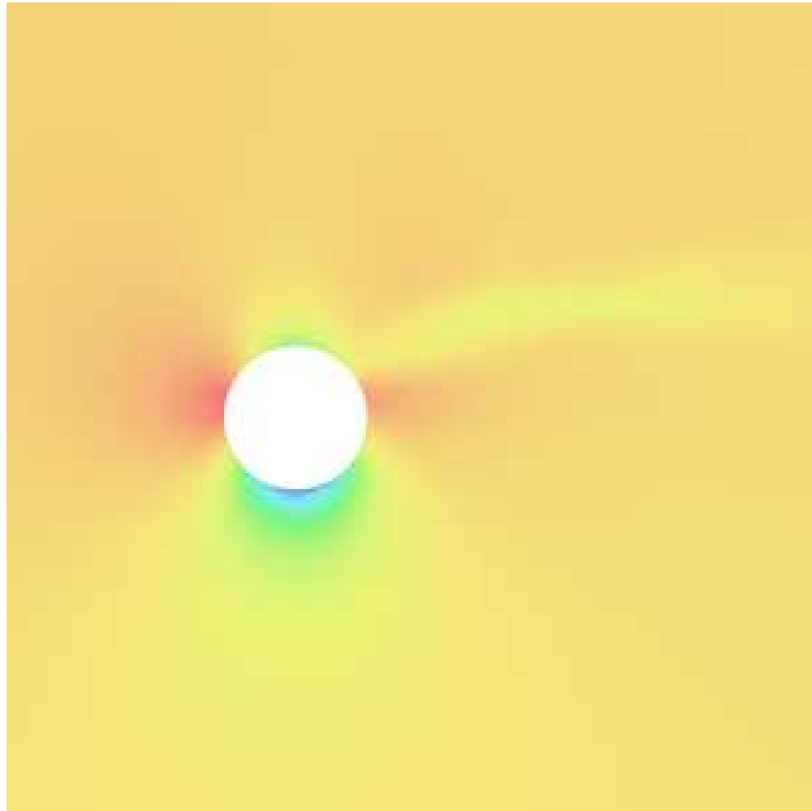


Velocity & pressure: t=2.0: $c_D = 0.51$

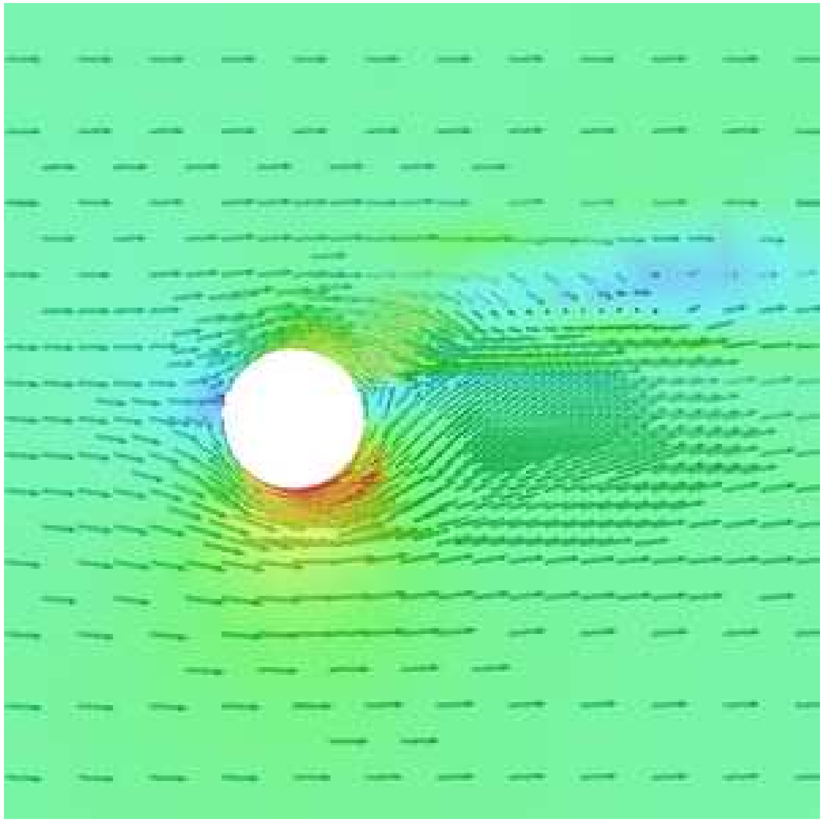


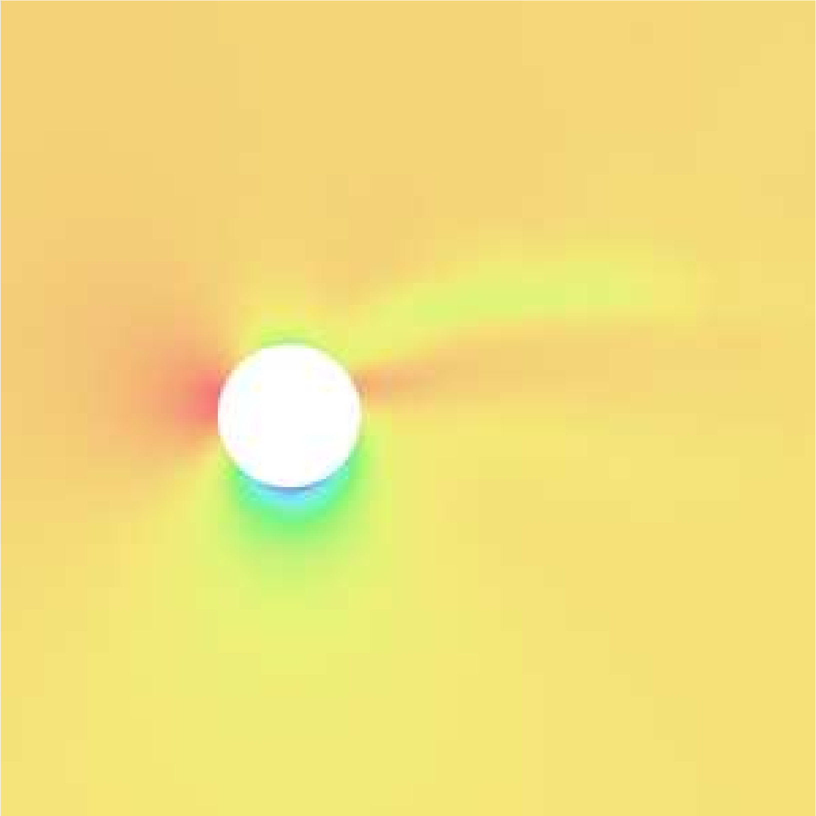
Velocity & pressure: t=2.25: $c_D = 0.78$



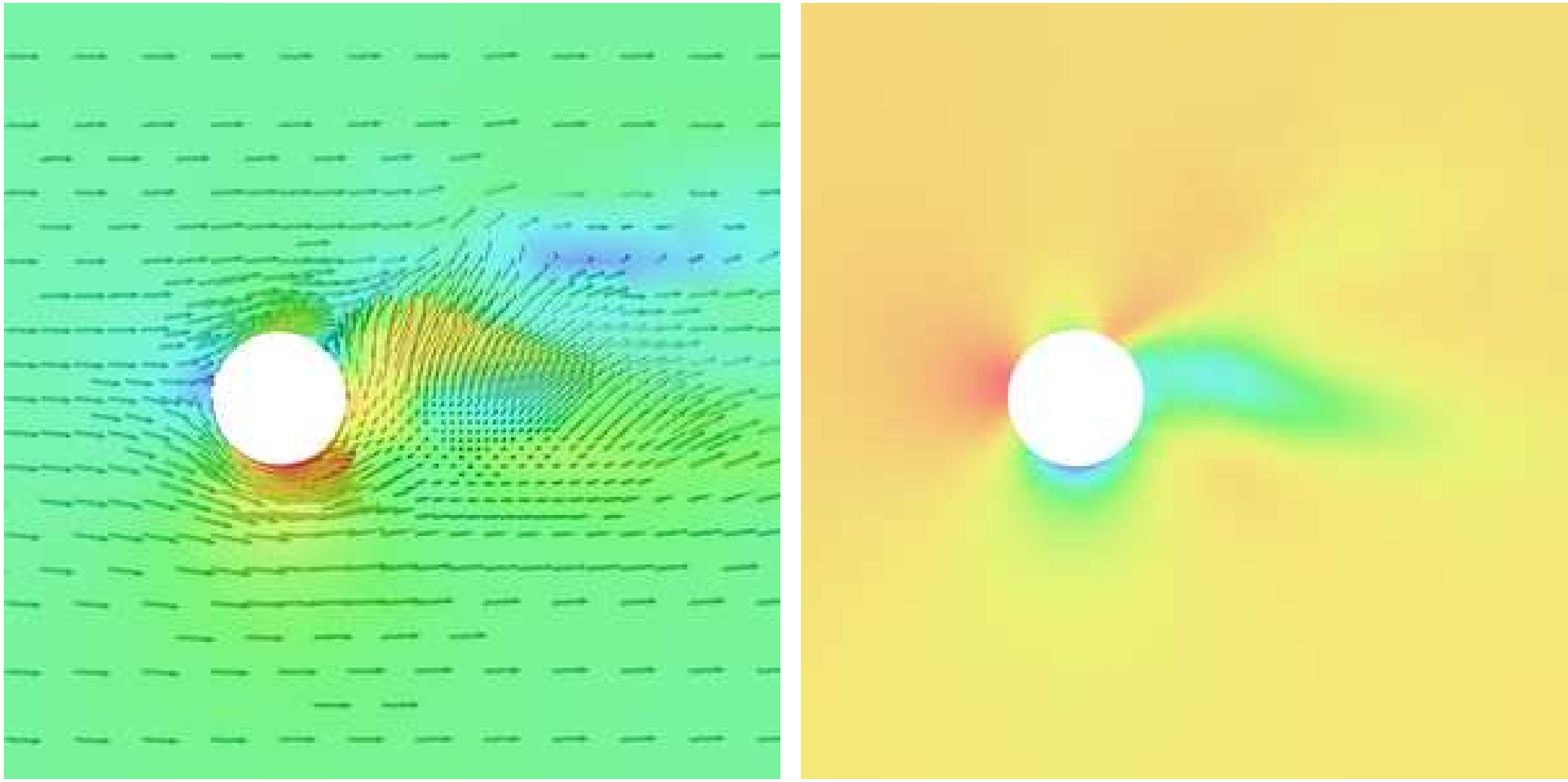


Velocity & pressure: t=2.5: $c_D = 1.14$

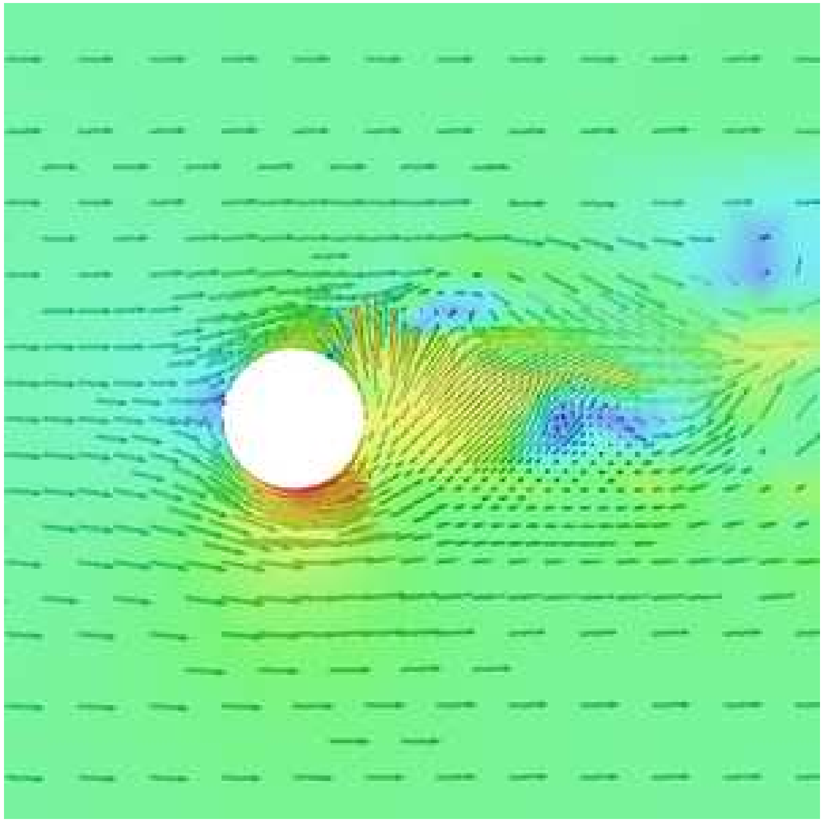


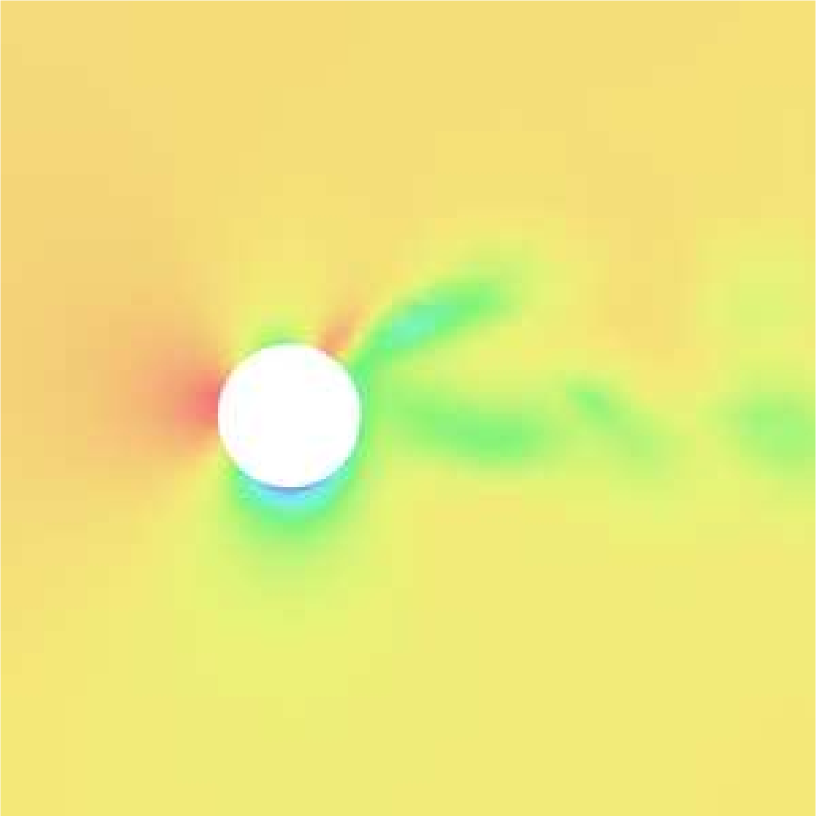


Velocity & pressure: t=2.75: $c_D = 1.04$

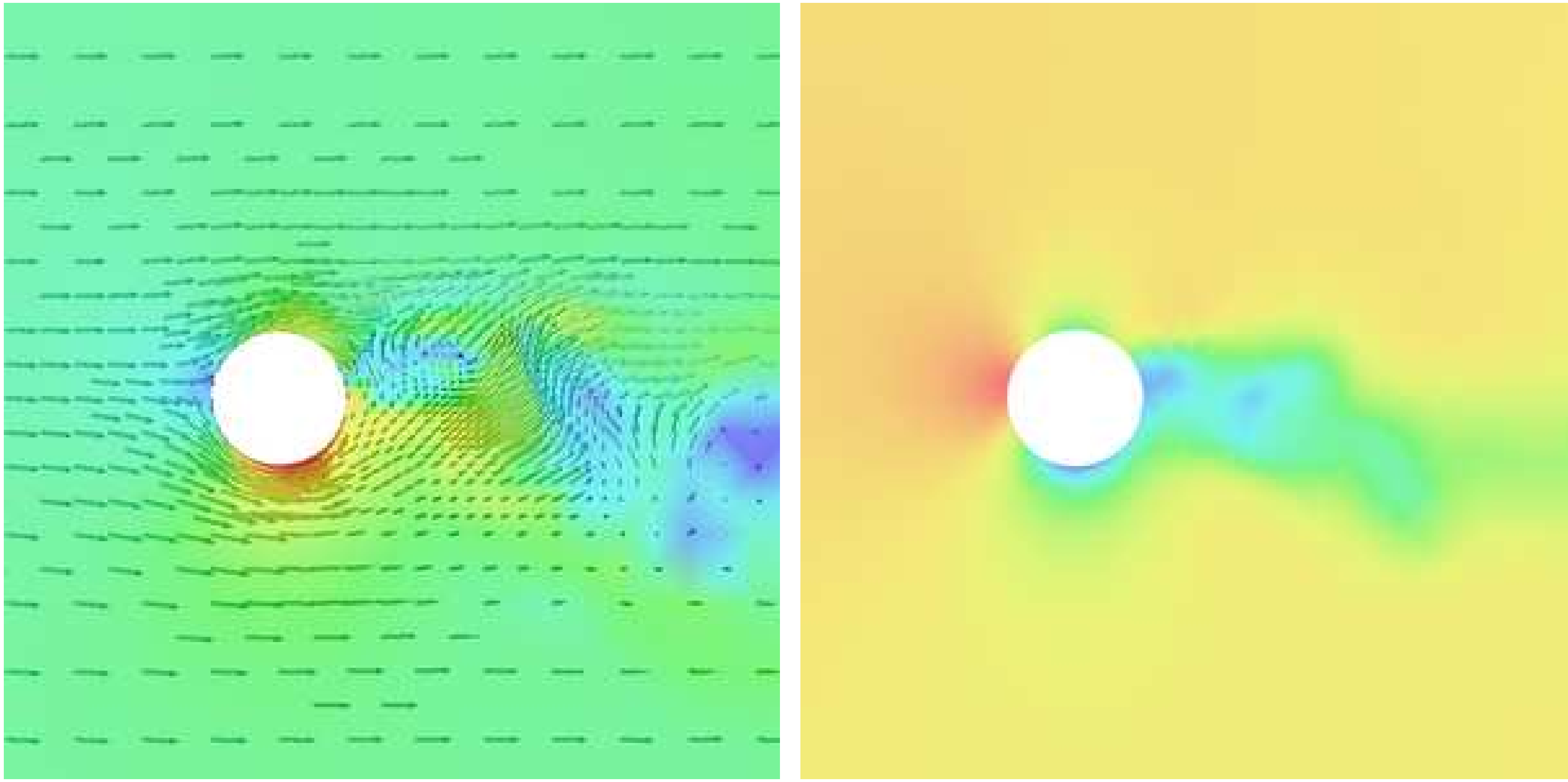


Velocity & pressure: t=3.0: $c_D = 1.05$

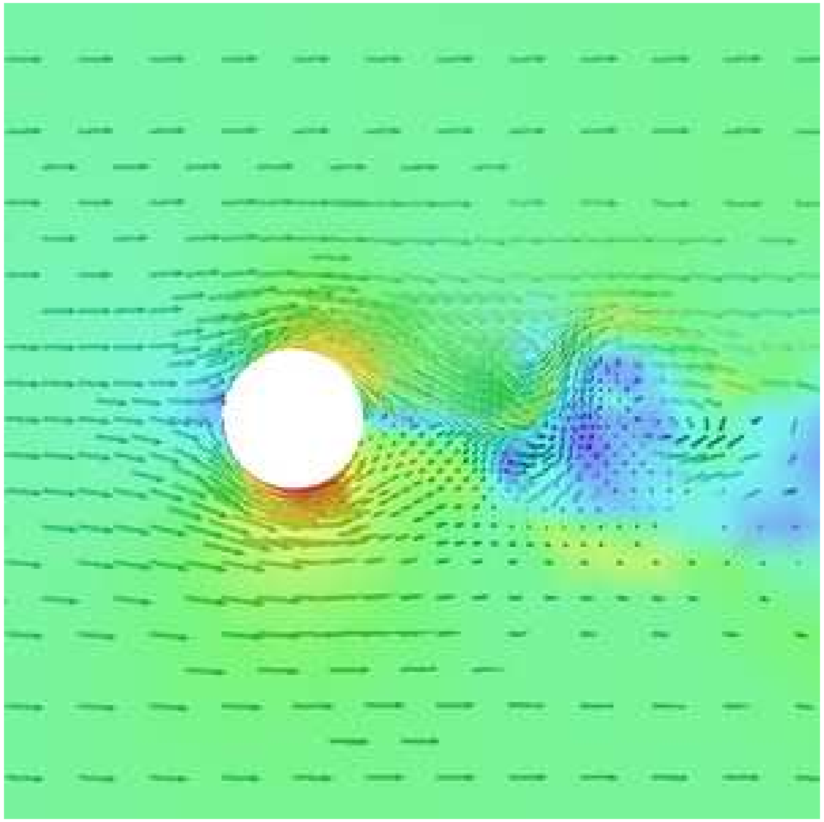
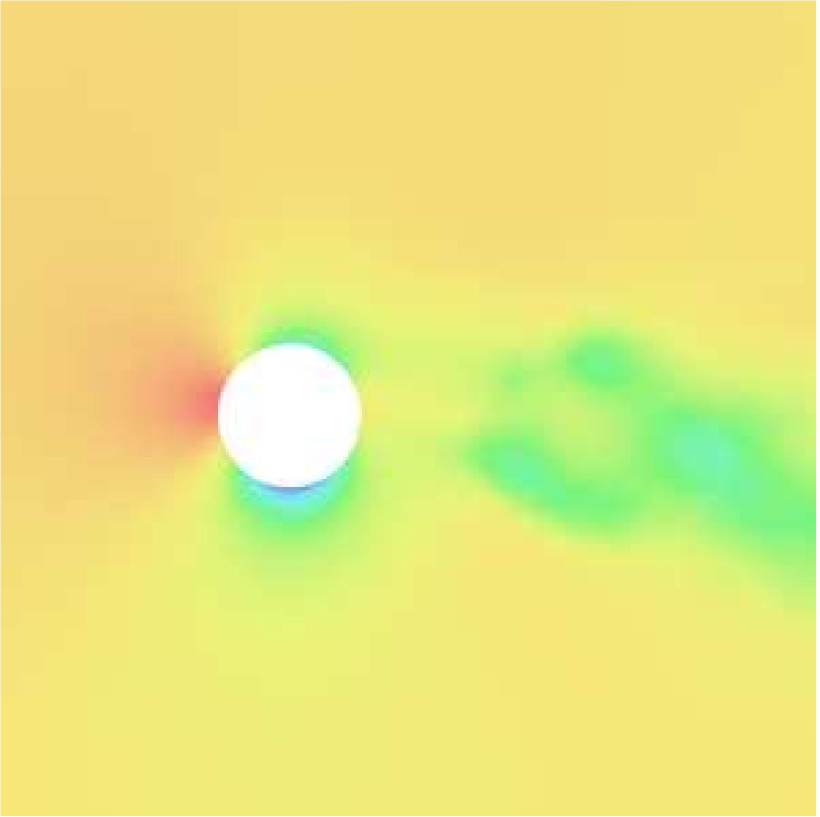




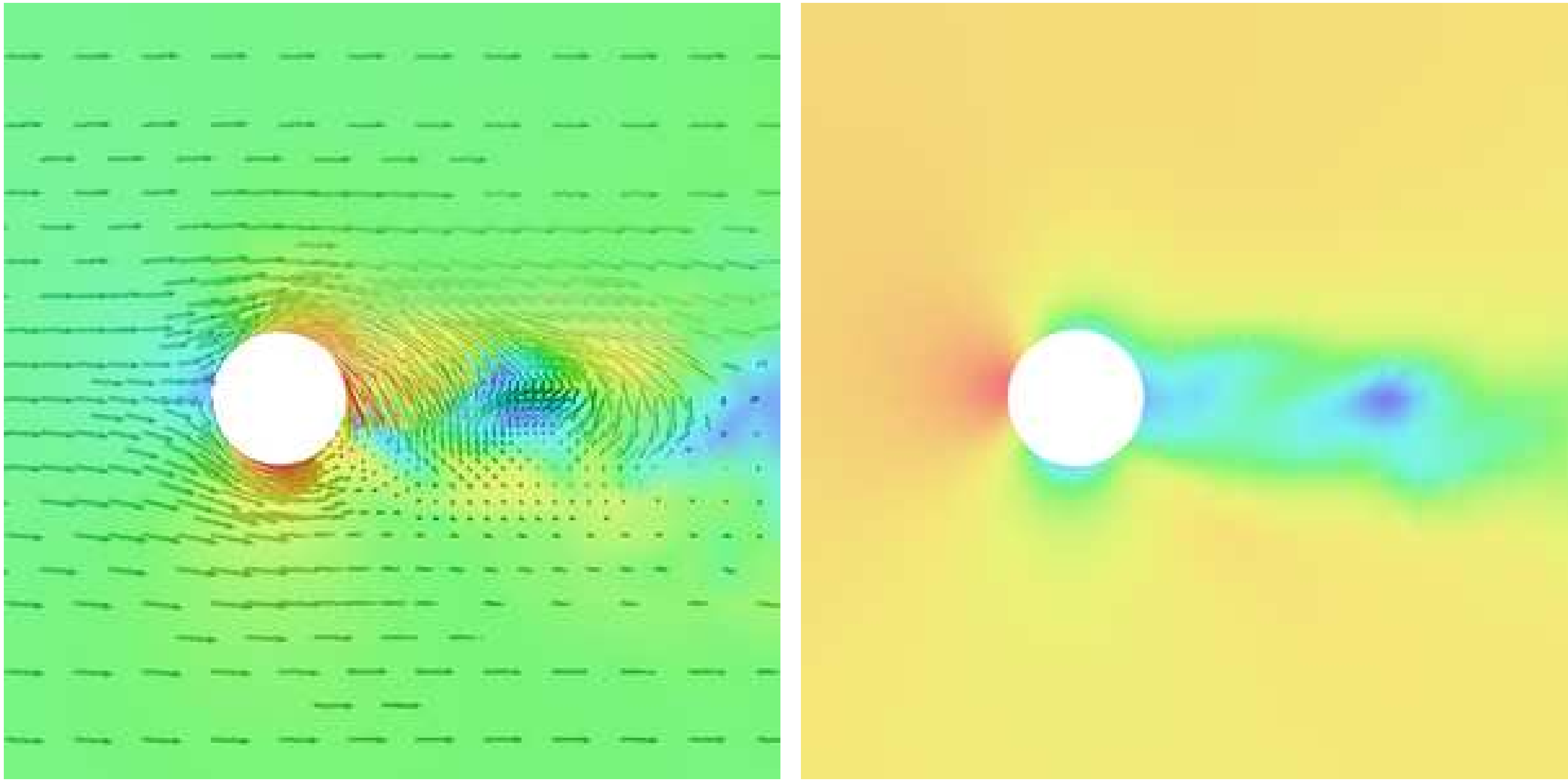
Velocity & pressure: t=4.5: $c_D = 1.63$



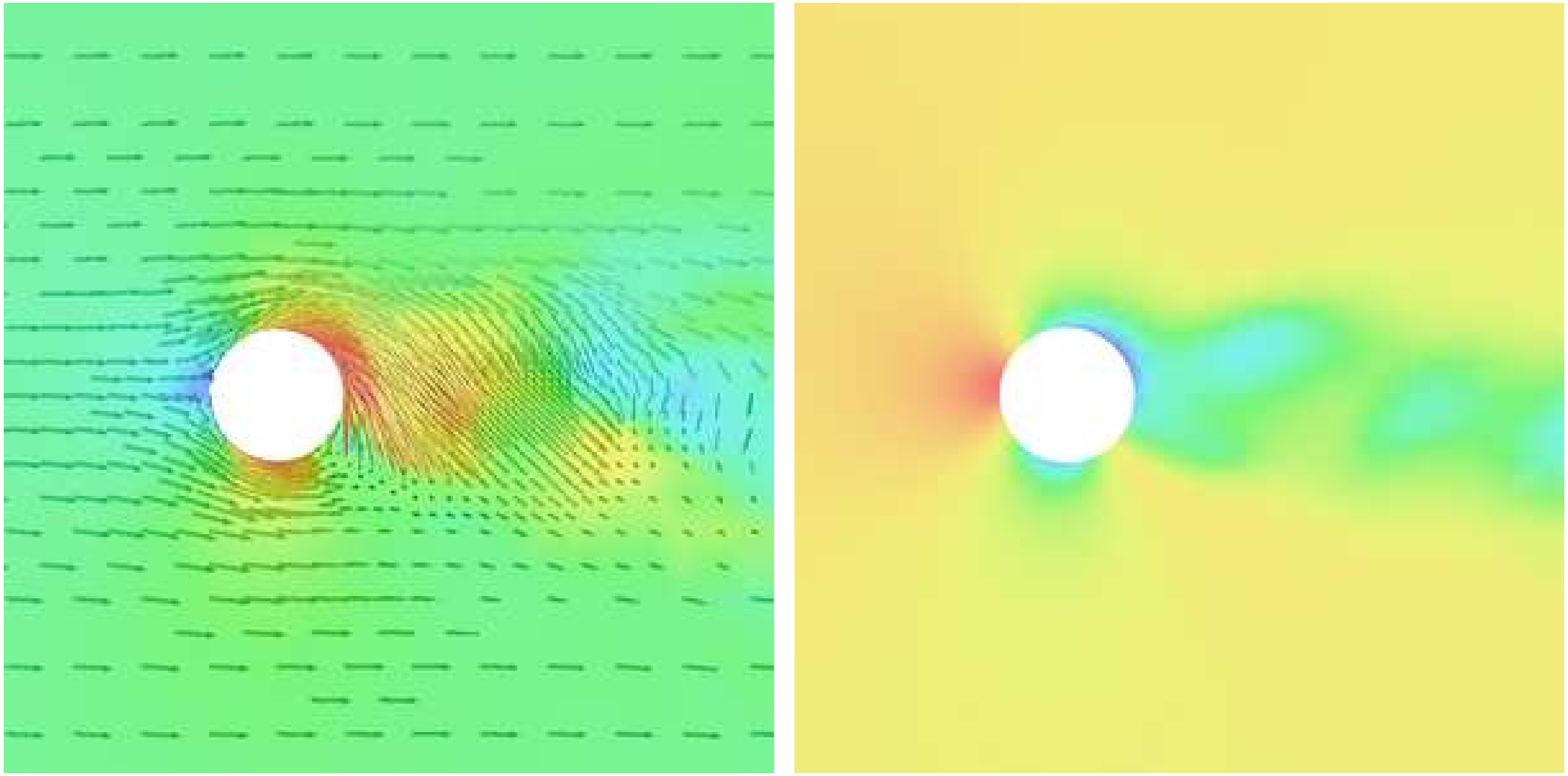
Velocity & pressure: t=5.0: $c_D = 1.79$

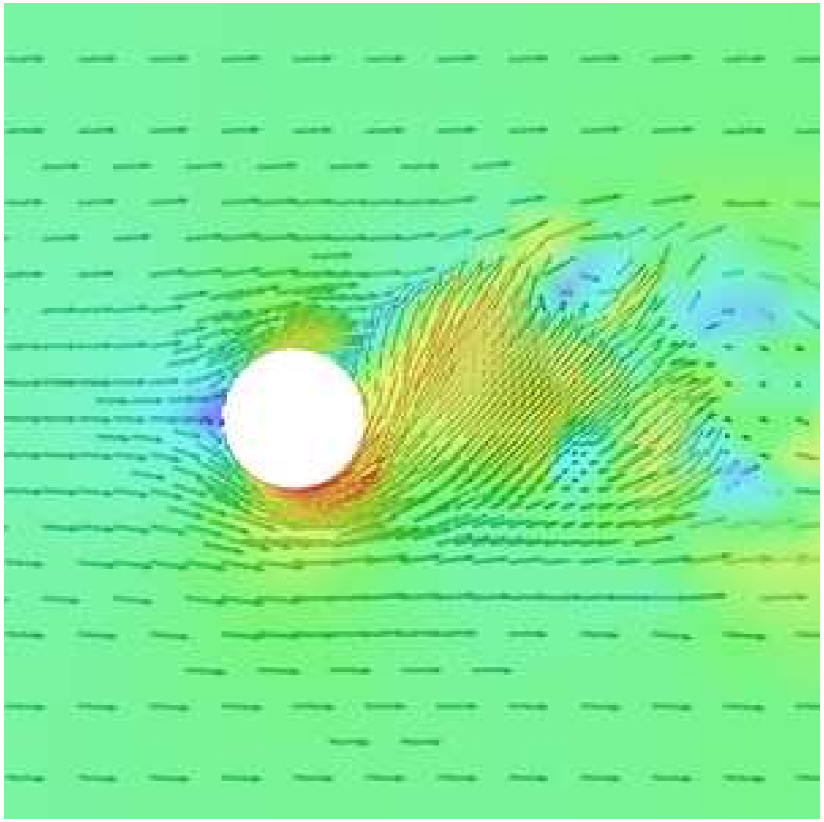
Velocity & pressure: t=5.5: $c_D = 1.96$

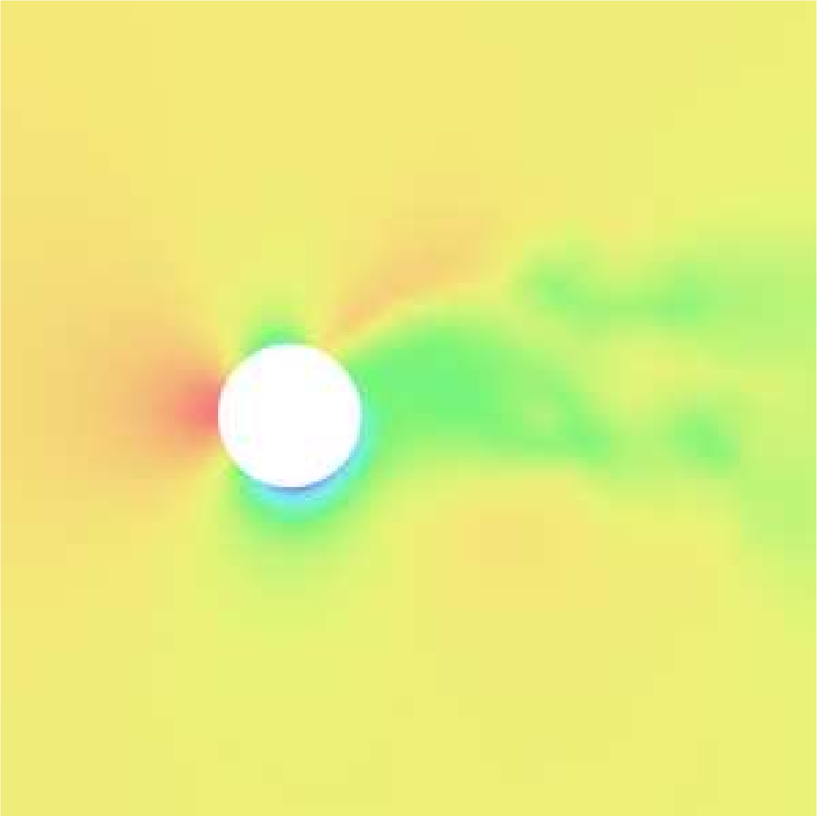


Velocity & pressure: t=5.75: $c_D = 1.90$



Velocity & pressure: t=11.0: $c_D = 1.82$

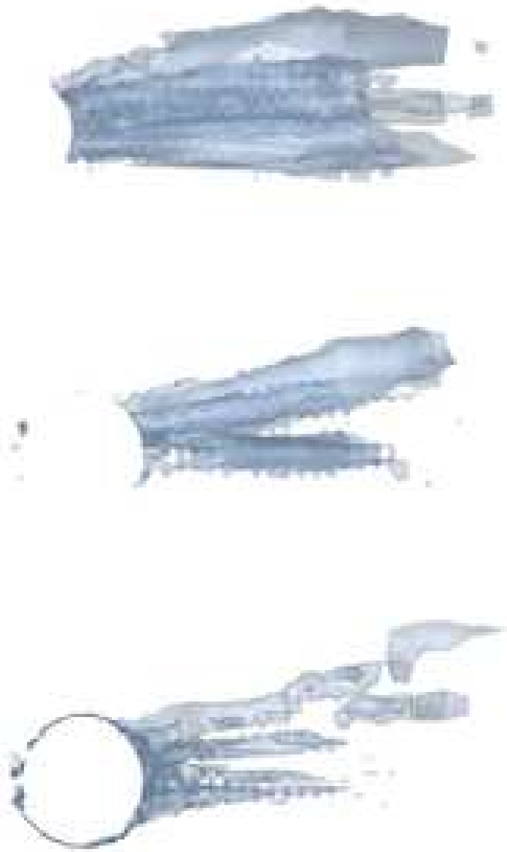




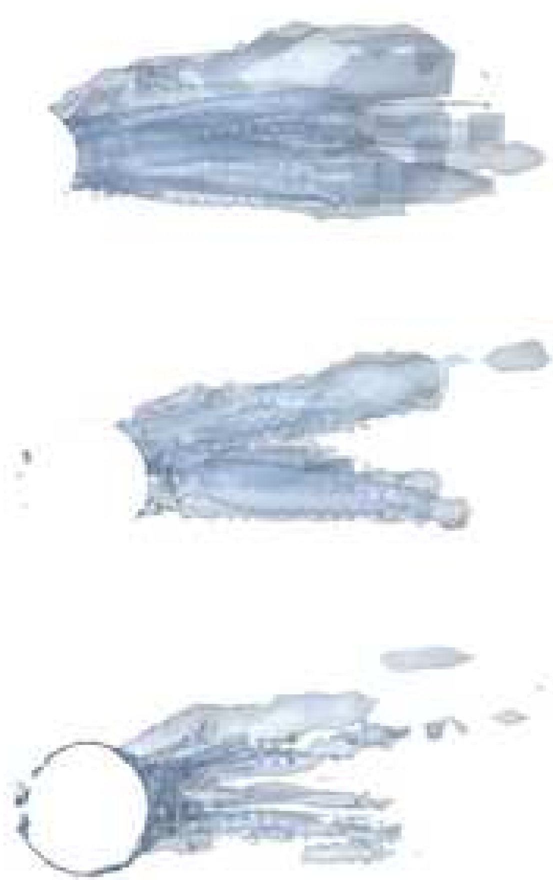
Vorticity: ω_1, ω_2, ω_3: t=1.0: $c_D = 0.22$

Vorticity: ω_1, ω_2, ω_3: t=1.25: $c_D = 0.25$

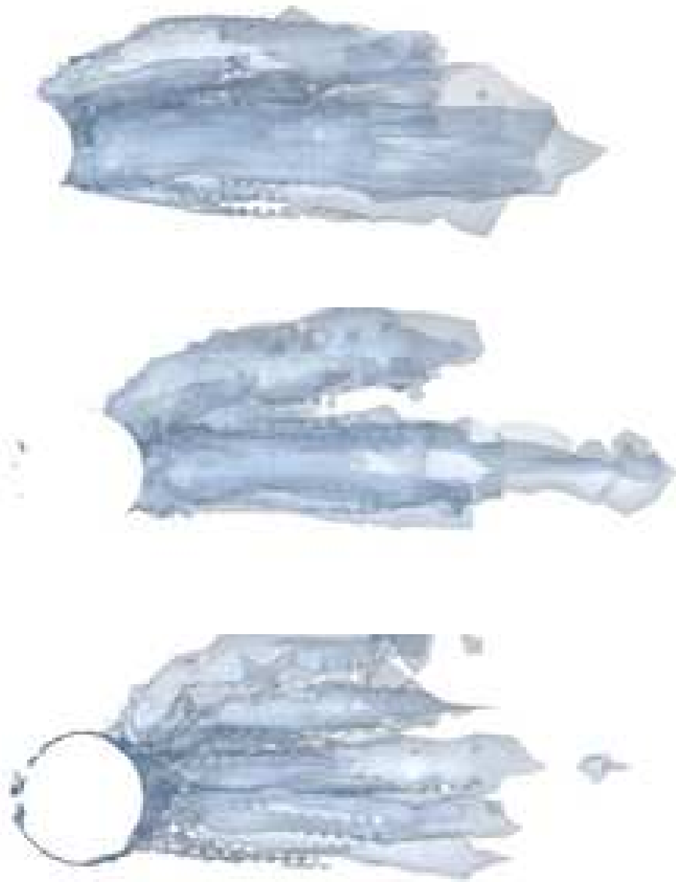
Vorticity: ω_1, ω_2, ω_3: t=1.5: $c_D = 0.28$



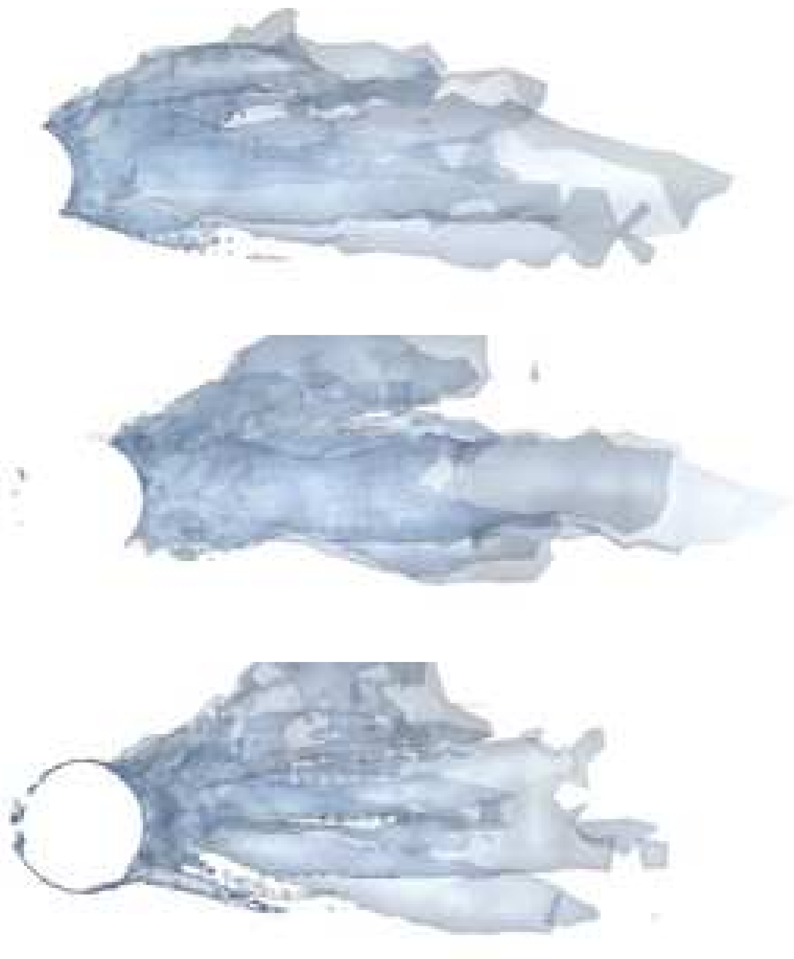
Vorticity: ω_1, ω_2, ω_3: t=1.75: $c_D = 0.36$



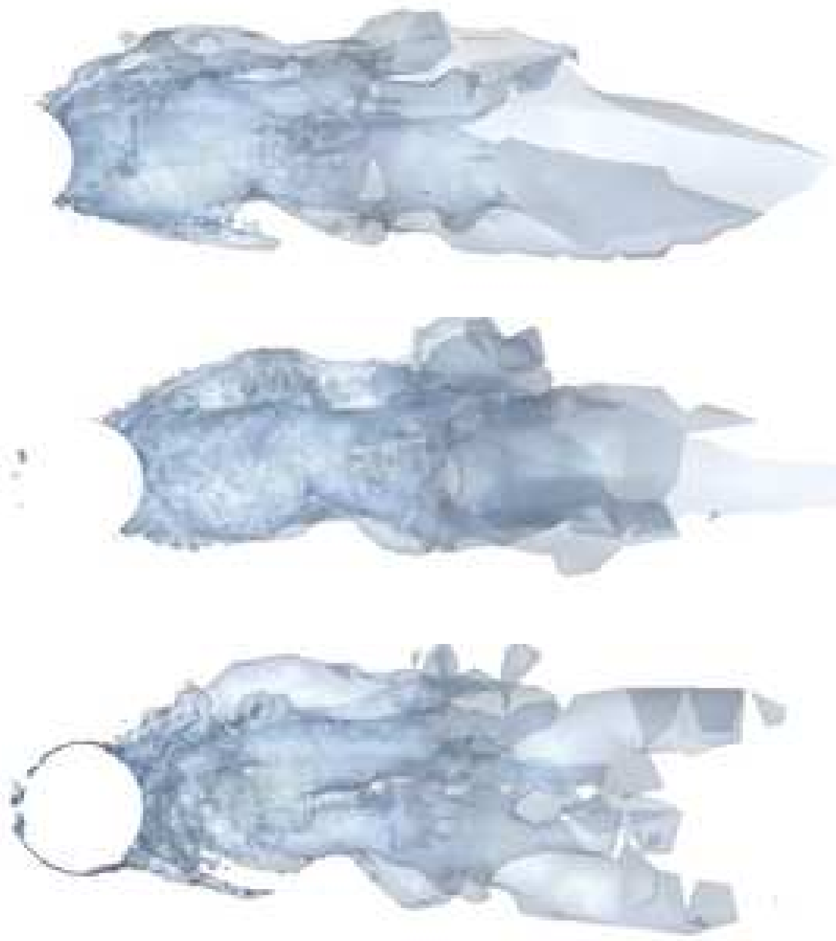
Vorticity: ω_1, ω_2, ω_3: t=2.25: $c_D = 0.78$



Vorticity: ω_1, ω_2, ω_3: t=2.5: $c_D = 1.14$



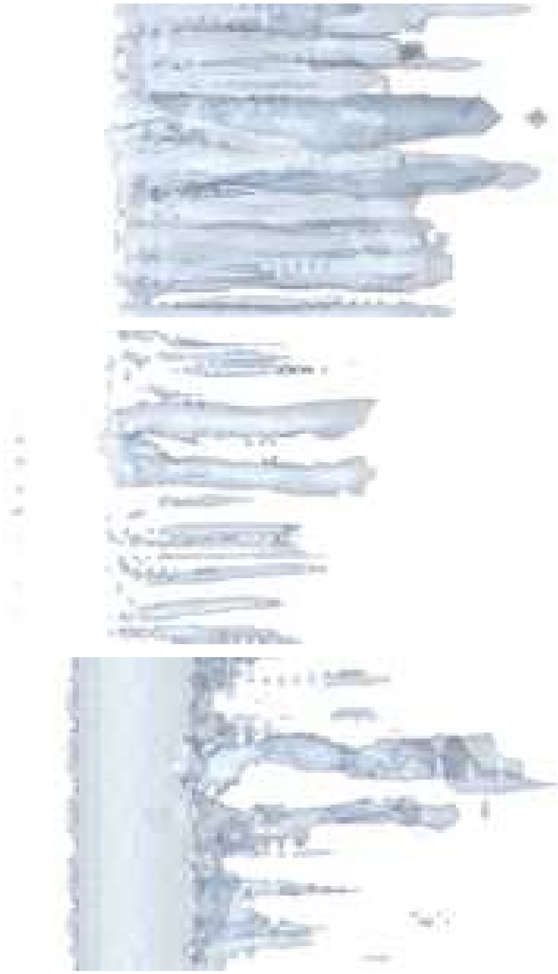
Vorticity: ω_1, ω_2, ω_3: t=2.75: $c_D = 1.04$



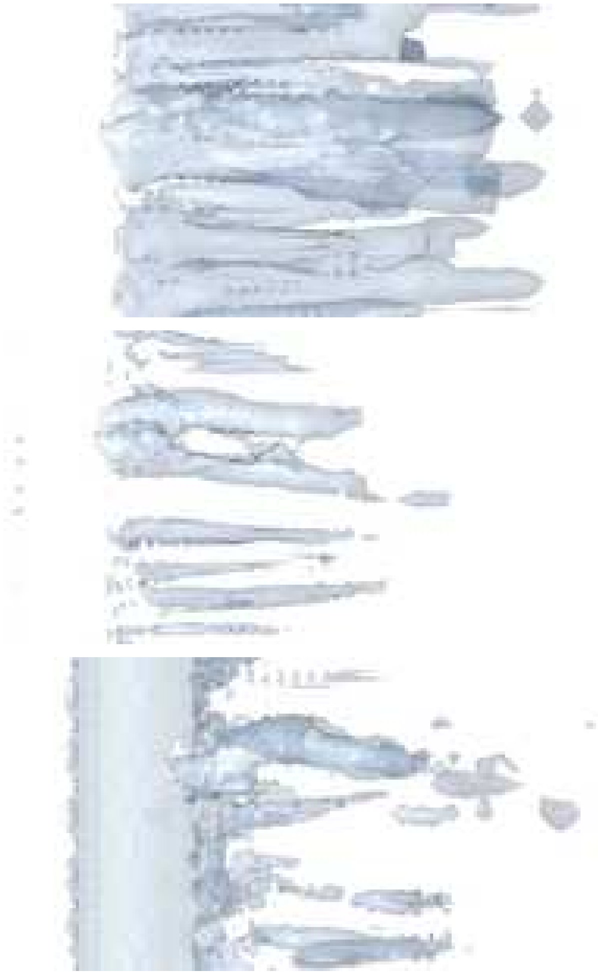
Vorticity: ω_1, ω_2, ω_3: t=1.0: $c_D = 0.22$

Vorticity: ω_1, ω_2, ω_3: t=1.25: $c_D = 0.25$

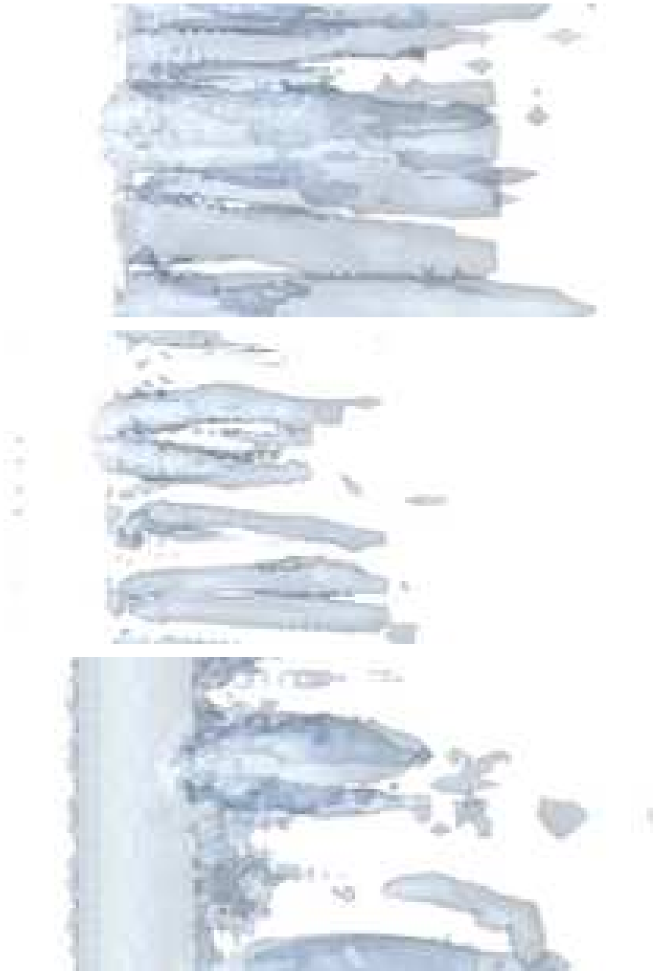
Vorticity: ω_1, ω_2, ω_3: t=1.5: $c_D = 0.28$



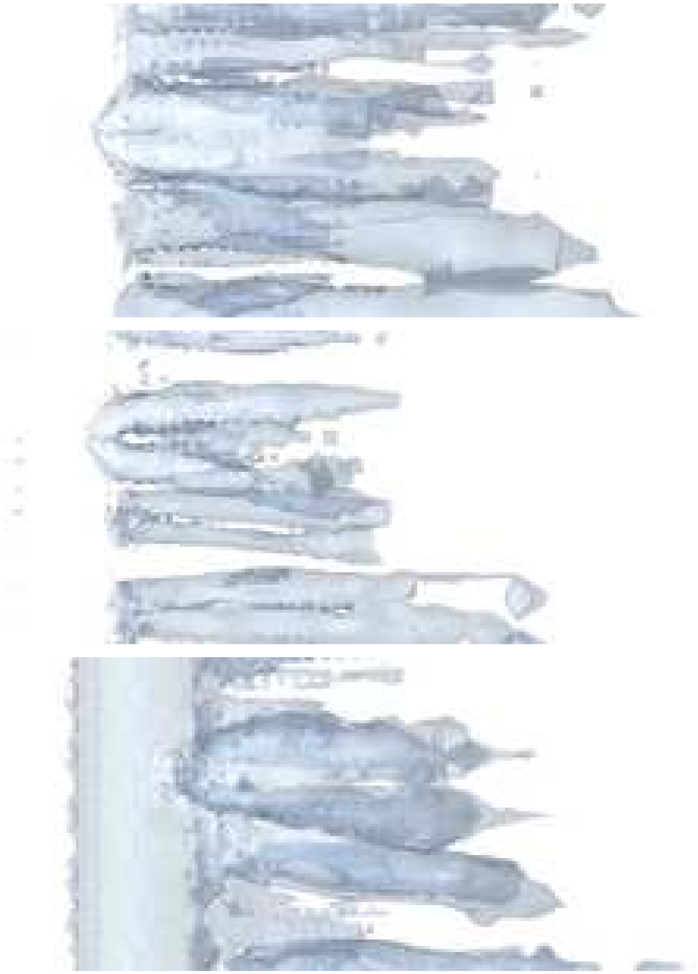
Vorticity: ω_1, ω_2, ω_3: t=1.75: $c_D = 0.36$



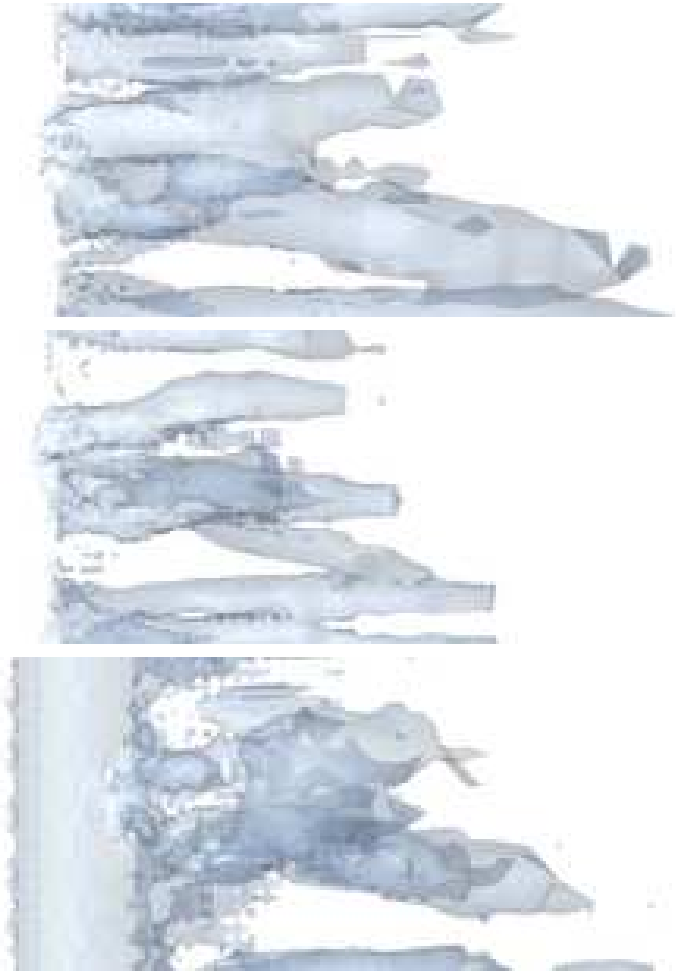
Vorticity: ω_1, ω_2, ω_3: t=2.0: $c_D = 0.51$



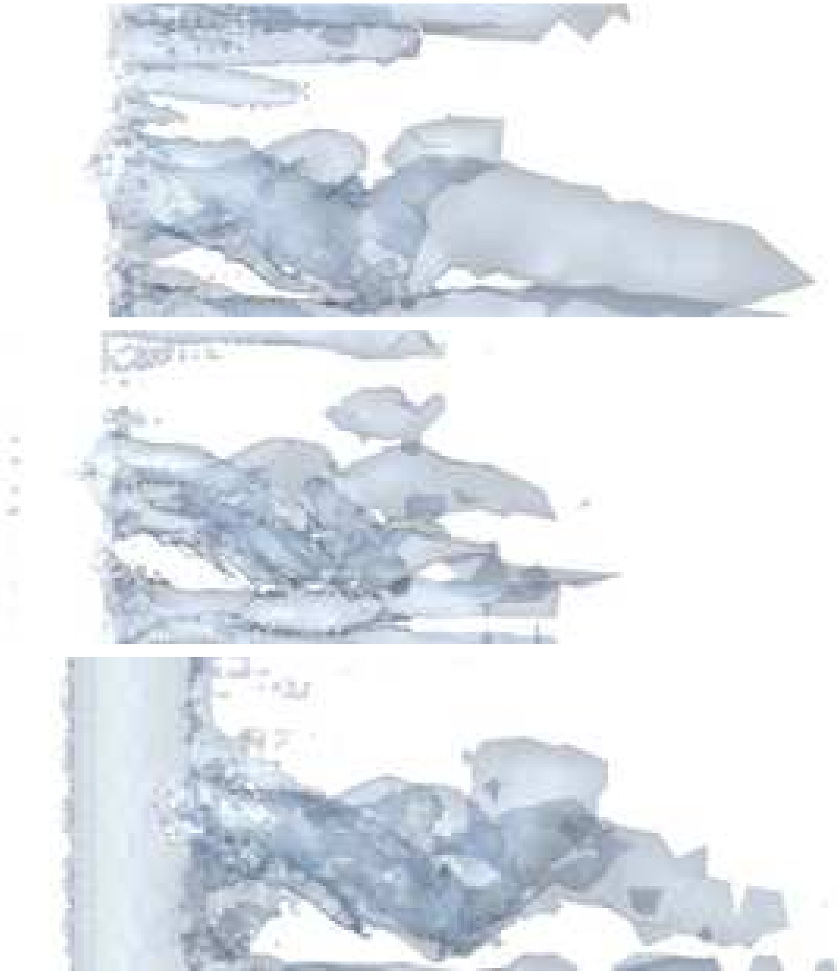
Vorticity: ω_1, ω_2, ω_3: t=2.25: $c_D = 0.78$



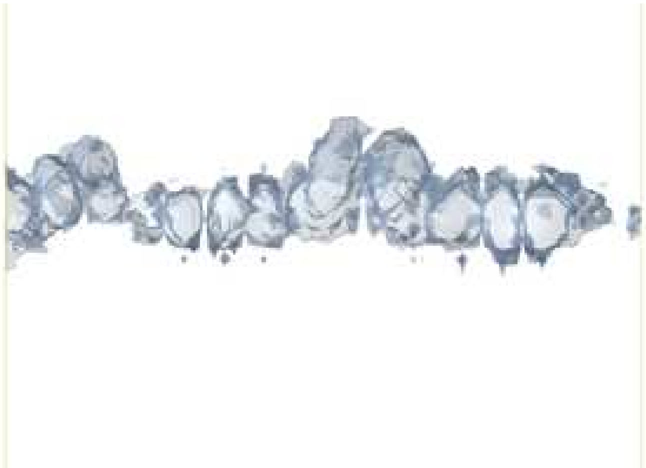
Vorticity: ω_1, ω_2, ω_3: t=2.5: $c_D = 1.14$



Vorticity: ω_1, ω_2, ω_3: t=2.75: $c_D = 1.04$

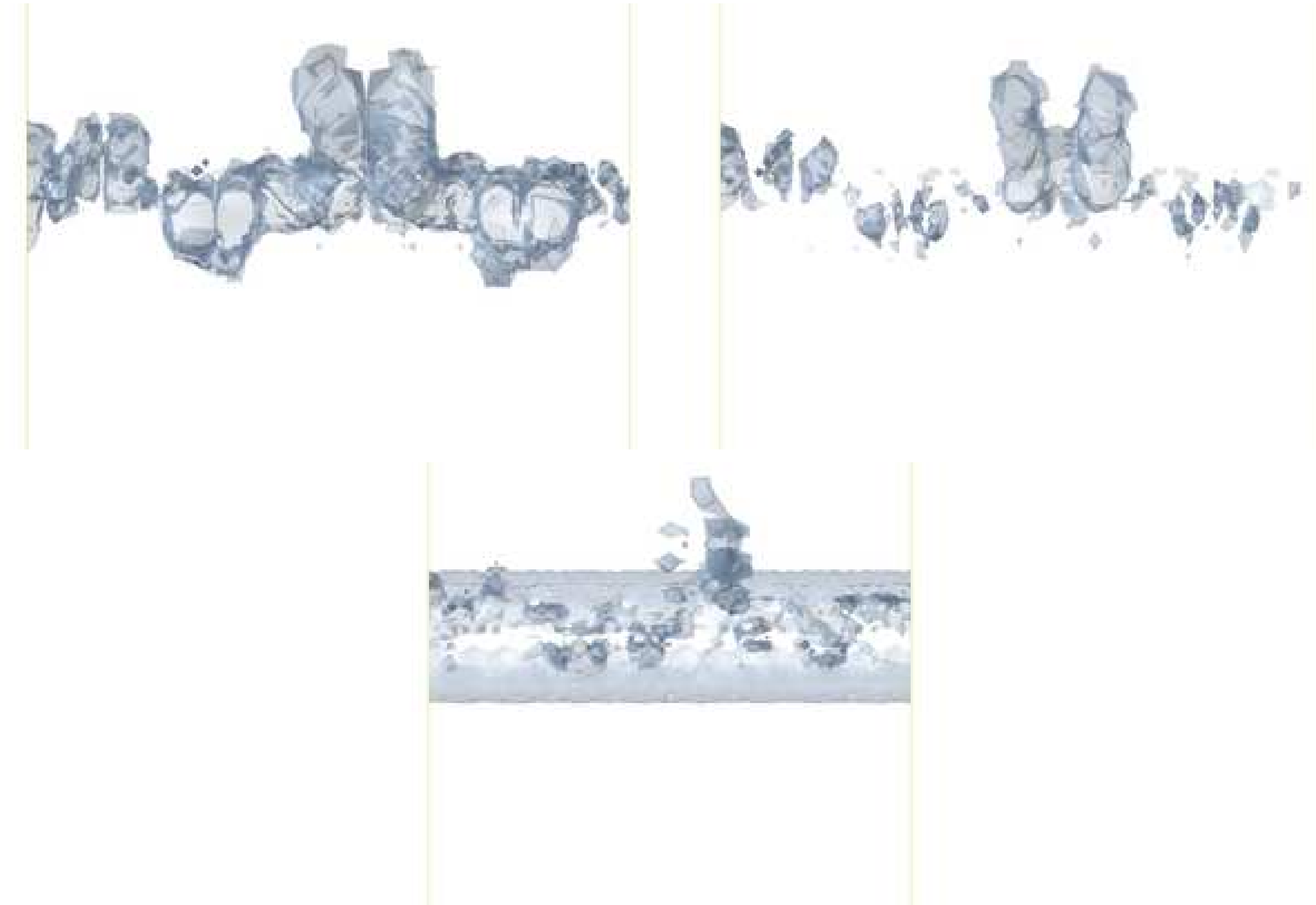


Vorticity: ω_1**,** ω_2**,** ω_3**: t=1.0:** $c_D = 0.22$

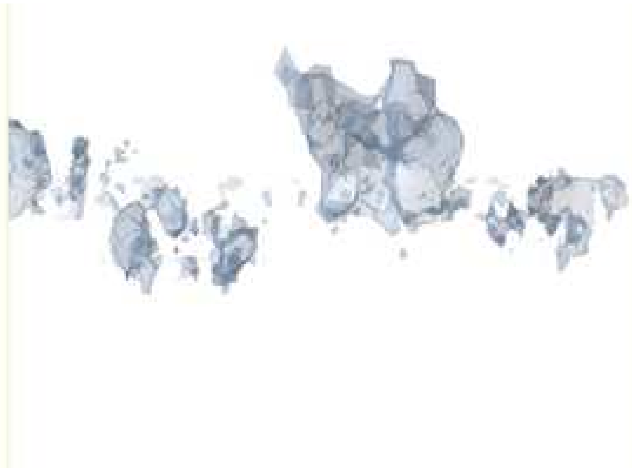


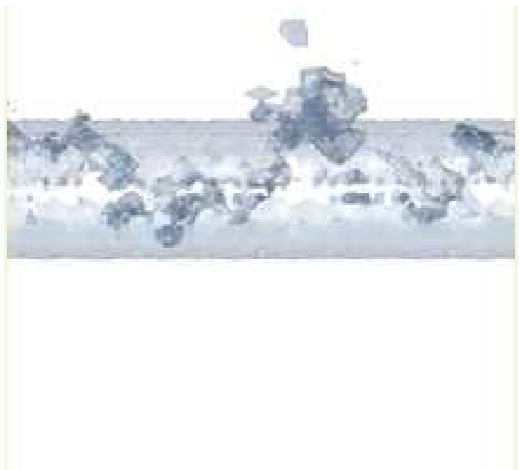
Vorticity: ω_1, ω_2, ω_3: t=1.25: $c_D = 0.25$

Vorticity: ω_1, ω_2, ω_3: t=1.5: $c_D = 0.28$

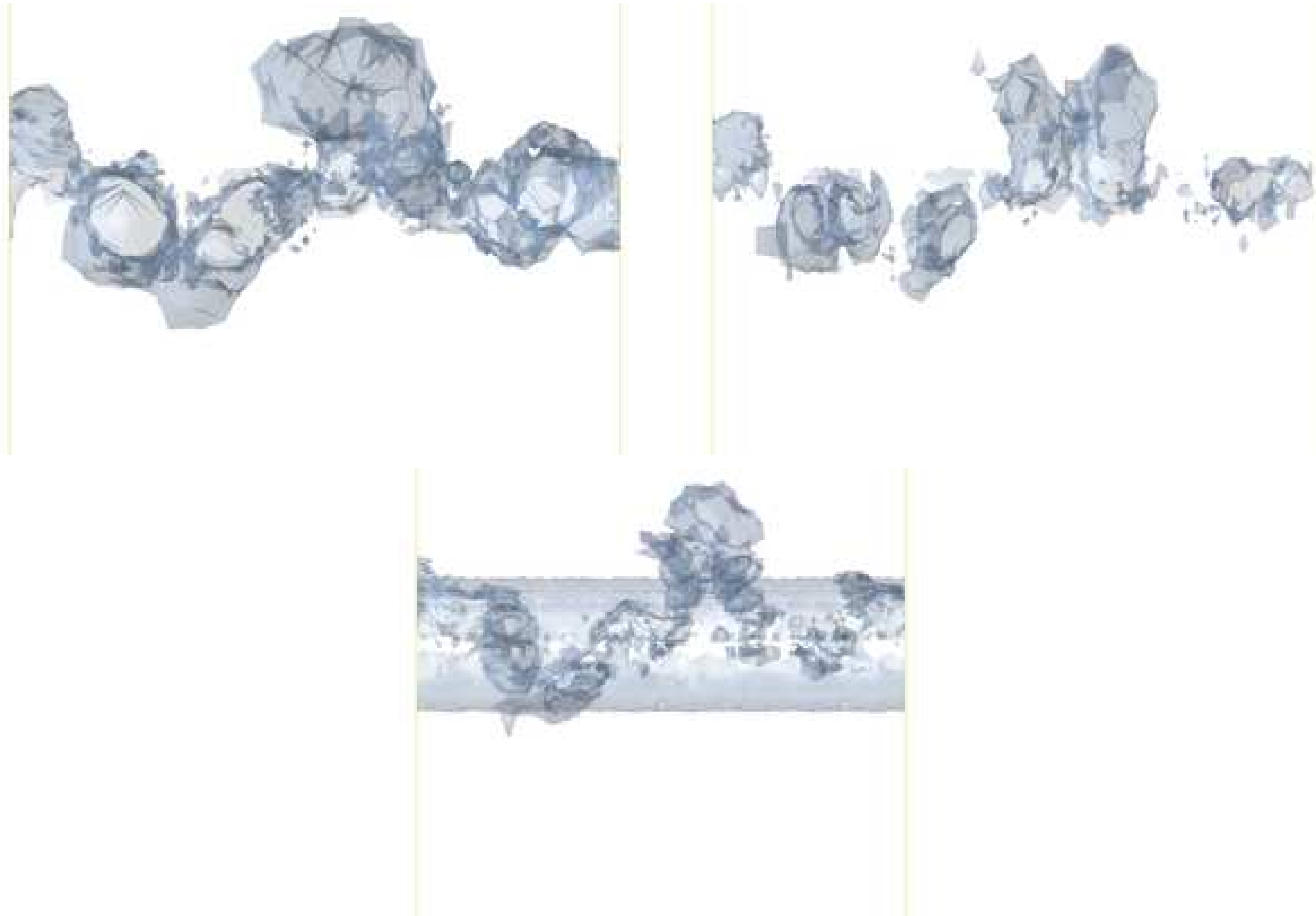


Vorticity: ω_1, ω_2, ω_3: t=1.75: $c_D = 0.36$

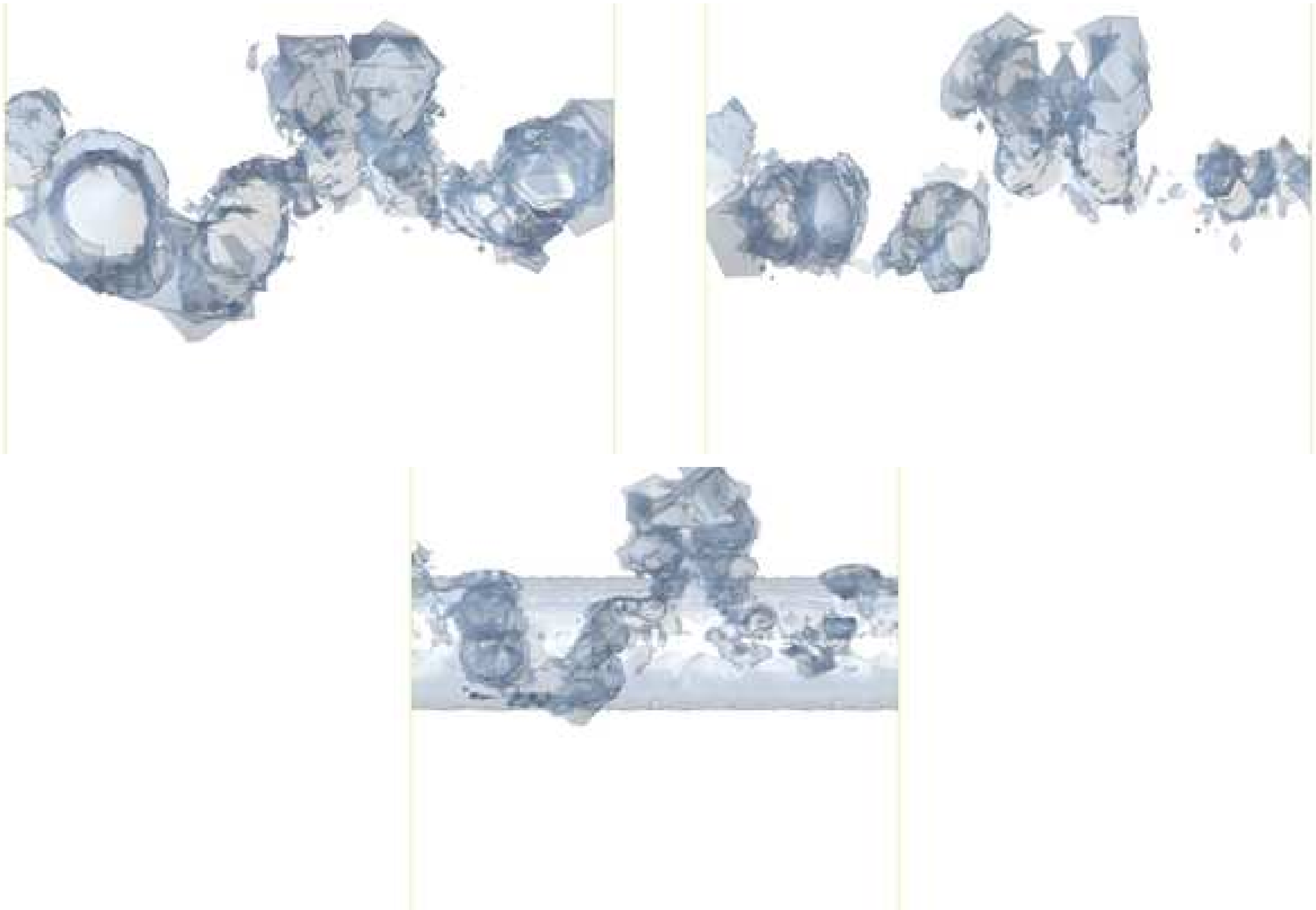




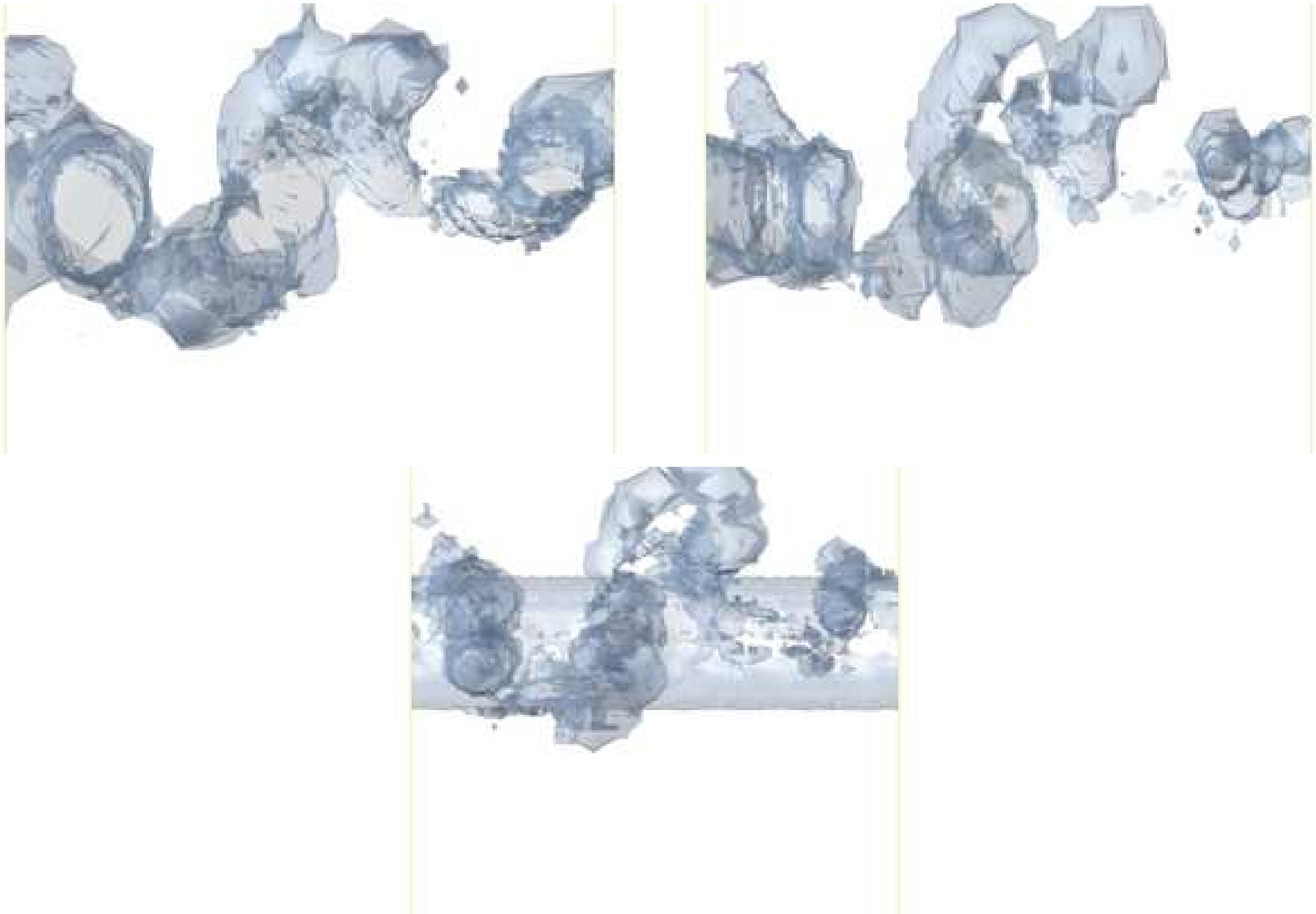
Vorticity: ω_1, ω_2, ω_3: **t=2.0:** $c_D = 0.51$



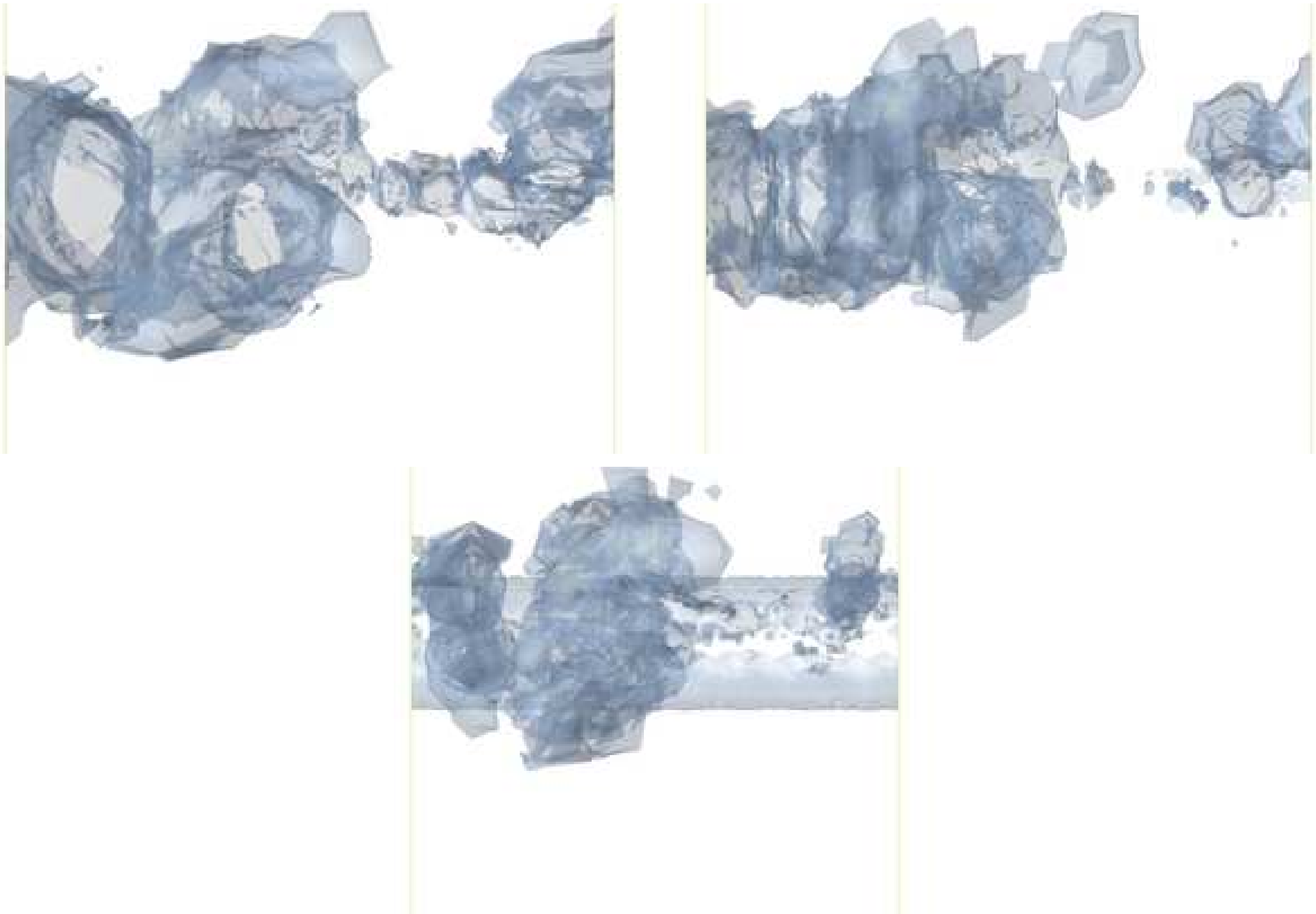
Vorticity: ω_1, ω_2, ω_3: t=2.25: $c_D = 0.78$



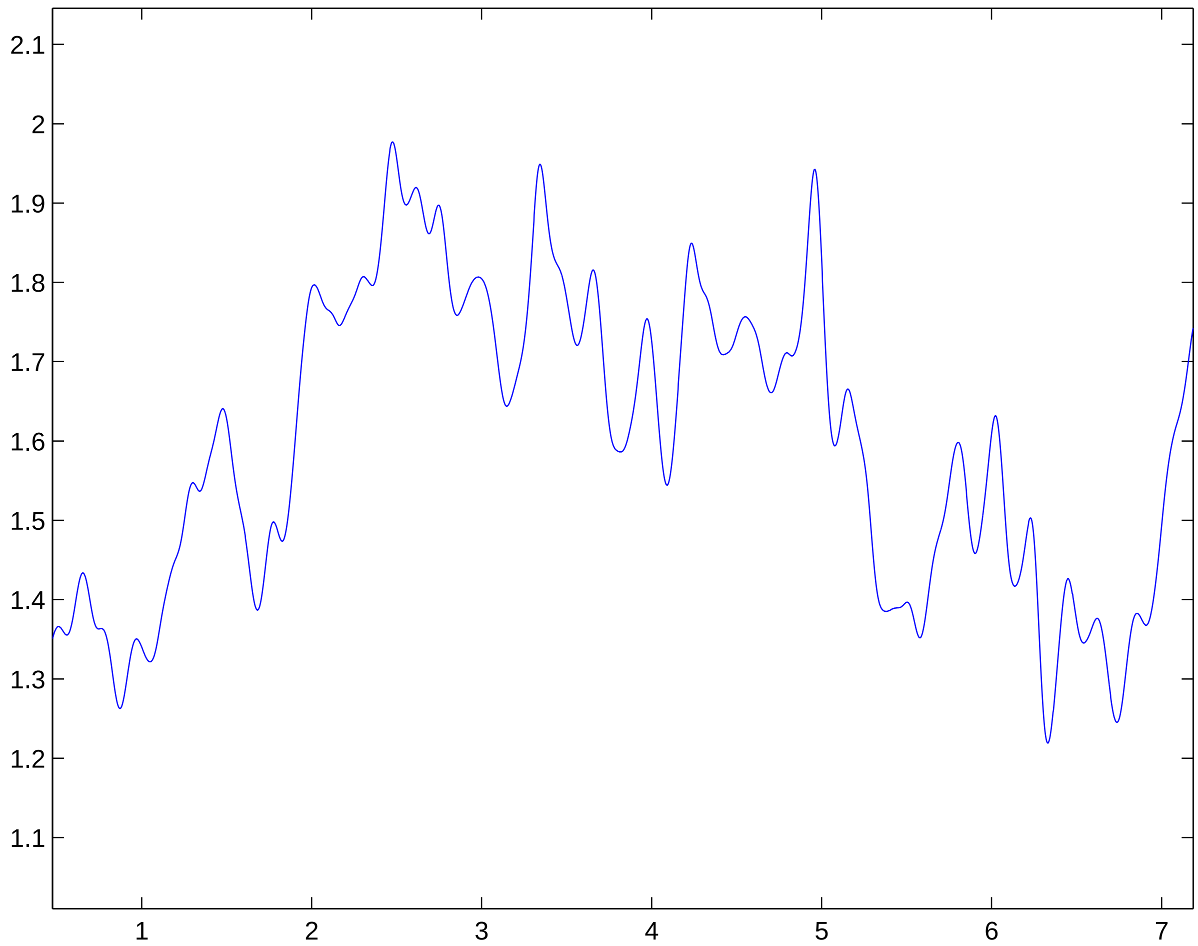
Vorticity: ω_1, ω_2, ω_3: t=2.5: $c_D = 1.14$



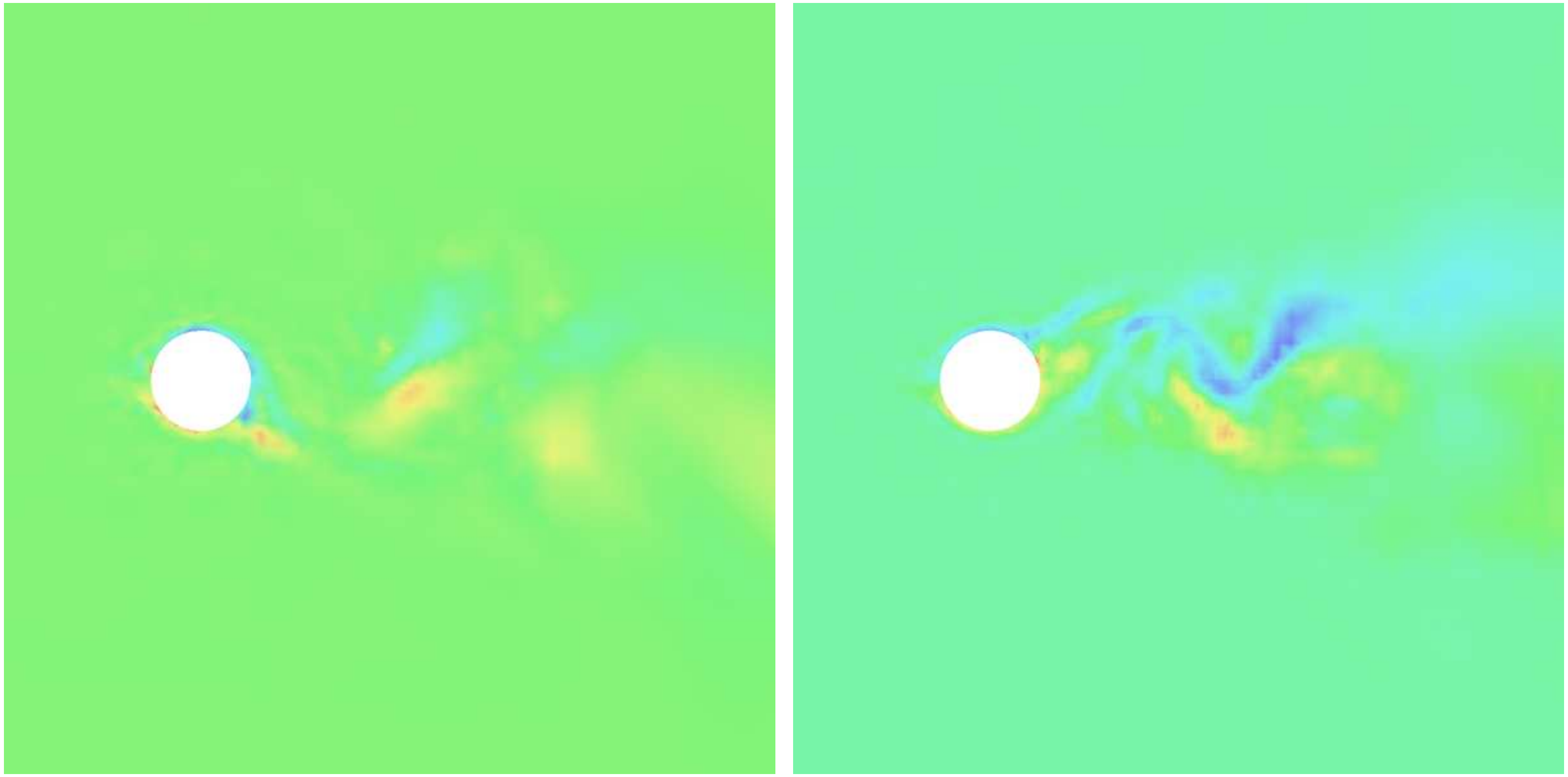
Vorticity: ω_1, ω_2, ω_3: t=2.75: $c_D = 1.04$



Oscillating EG2 solution: c_D

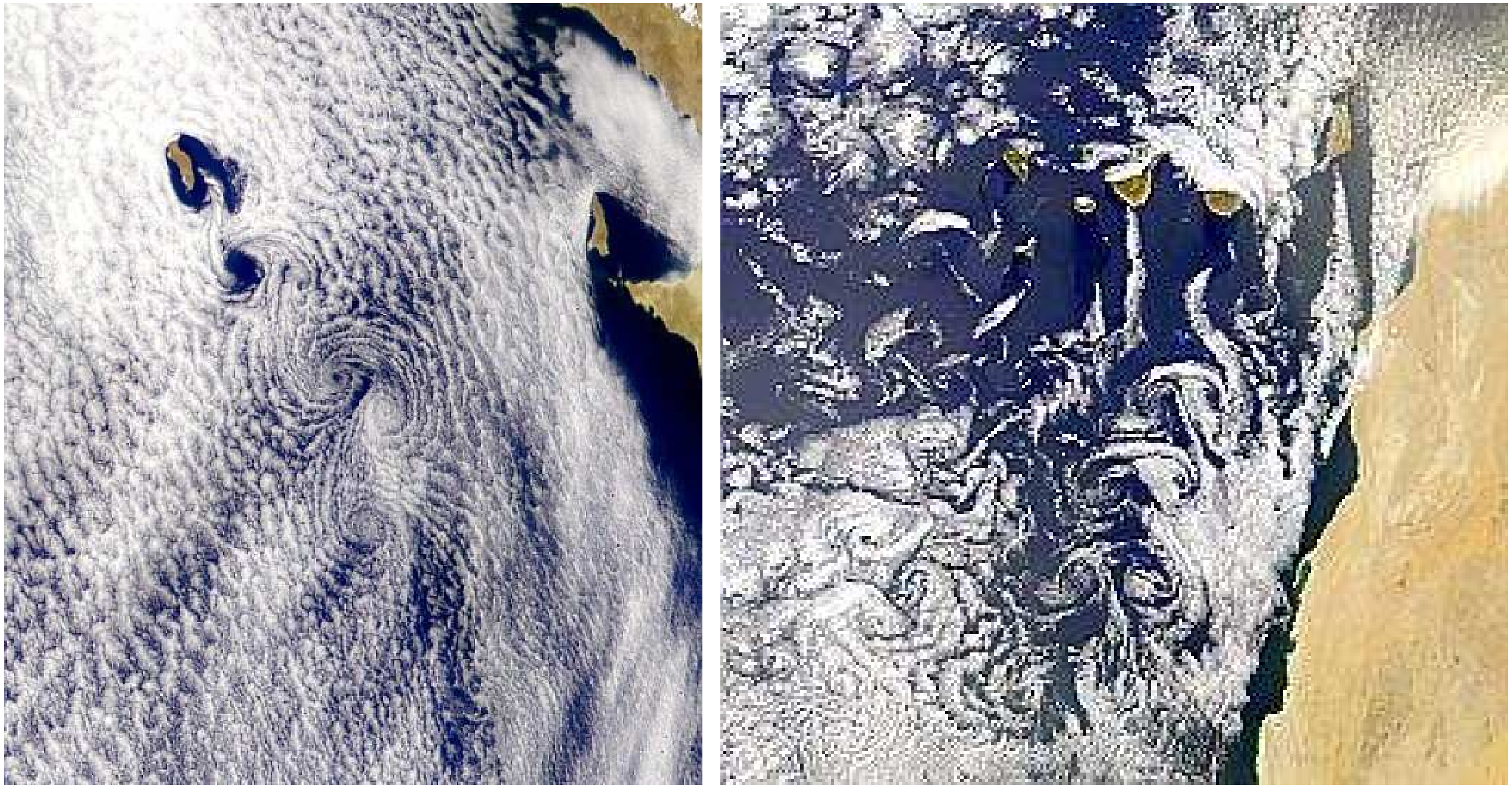


Oscillating EG2 solution



Note: there is only one separation point (in each section)

Guadalupe, Canaries: $Re \approx 10^9$



Note: only one separation point (consistent with EG2)

EG2 and Turbulent Euler solutions

- No experimental results for cylinder at $Re > 10^7$
- Therefore not much is known about this flow regime.
- For geophysical flow ($Re \approx 10^9$) we observe “von Karman vortex shedding”; but with separation in one point only (no wake!) consistent with the EG2 solution.
- EG2: no empirical parameters (only parameter is h)

“In a reasonable theory there are no dimensionless numbers whose values are only empirically determinable.” (Einstein)

Summary: NSE & G2

- Approximate weak solutions: existence by G2
- Weak uniqueness: duality: computation of $S_\epsilon(\hat{\psi})$
- Computational method: G2 (Adaptive DNS/LES)
  - Adaptivity $\rightarrow$ very low computational cost
  - Generality $\rightarrow$ quick & easy to adjust to new problem
  - Quantitative error control $\rightarrow$ reliable results
- No filtering $\Rightarrow$ no Reynolds stresses
- Turbulent boundary layer by simple friction b.c.
- EG2: parameter-free model of high Re flow