FEniCS Course

Overview and Introduction

Lecturer
Anders Logg
Beijing, August 5–7 2015

Course outline

	⋆	Overview and Introduction
Wed	**L01**	Installation of FEniCS
	L02	Static linear PDEs
	L04	Time-dependent PDEs
Thu	**L06**	Static hyperelasticity
	L07	Dynamic hyperelasticity
Fri	**L08**	The Stokes problem
	L09	Incompressible Navier–Stokes

Full list of FEniCS lectures

- ⋆ Overview and Introduction
- **L00** Introduction to FEM
- **L01** Installation of FEniCS
- **L02** Static linear PDEs
- **L03** Static nonlinear PDEs
- **L04** Time-dependent PDEs
- **L05** Happy hacking: Tools, tips and coding practices
- **L06** Static hyperelasticity
- **L07** Dynamic hyperelasticity
- **L08** The Stokes problem
- **L09** Incompressible Navier–Stokes
- **L10** Discontinuous Galerkin methods for elliptic equations
- **L11** A posteriori error estimates and adaptivity
- **L12** Computing sensitivities
- **L13** Introduction to dolfin-adjoint
- **L14** From sensitivities to optimisation
- **L14** One-shot optimisation
- **L16** Optimal control of the Navier-Stokes equations

All lectures can be downloaded from

`http://fenicsproject.org/pub/course/`

Key distinguishing features

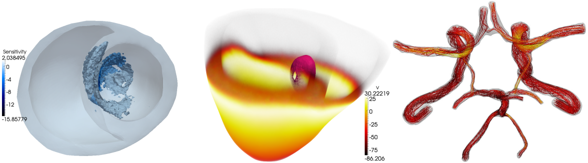
The FEniCS Project is a collection of open-source software components aimed at the numerical solution of partial differential equations using finite element methods

- FEniCS (Python/C++) code is quick to write and easy to read
- 'Any' finite element formulation of 'any' partial differential equation can be coded
- Automated code generation is heavily used under the hood to create efficient, specialized, low-level code
- Performance – implicit problems with over 12 000 000 000 degrees of freedom can be solved in a couple of minutes

http://fenicsproject.org/

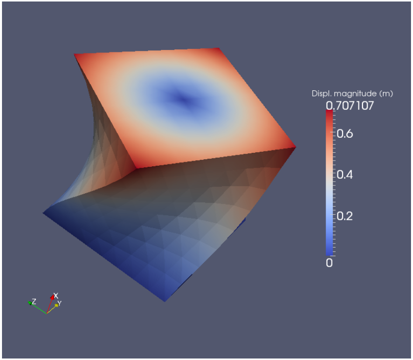
FEniCS has been used for a wide range of equations and applications

Reaction-diffusion equations; Stokes with or without nonlinear viscosity; compressible and incompressible Navier–Stokes; RANS turbulence models; shallow water equations; Bidomain equations; nonlinear and linear elasticity; nonlinear and linear viscoelasticity; Schrödinger; Biot's equations for porous media, fracture mechanics, electromagnetism, liquid crystals including liquid crystal elastomers, combustion, ... and coupled systems of the above, ...



for simulating blood flow, computing calcium release in cardic tissue, computing the cardiac potential in the heart, simulating mantle convection, simulating melting ice sheets, computing the optimal placement of tidal turbines, simulating and reconstructing tsunamis, simulating the flow of cerebrospinal fluid and the deformation of the spinal cord, simulating waveguides, ...

Hyperelasticity

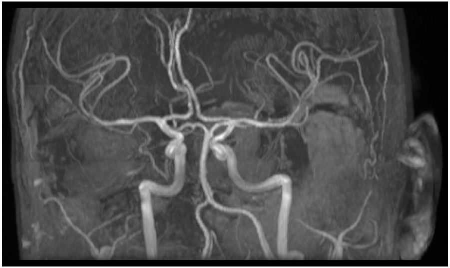


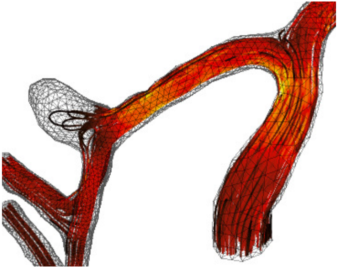
Python code

```
from fenics import *

mesh = UnitCubeMesh(24, 16, 16)
V = VectorFunctionSpace(mesh, "Lagrange", 1)

left =  CompiledSubDomain("(std::abs(x[0])
    < DOLFIN_EPS) && on_boundary")
right = CompiledSubDomain("(std::abs(x[0] - 1.0)
    < DOLFIN_EPS) && on_boundary")

c = Expression(("0.0", "0.0", "0.0"))
r = Expression(("0.0",
"0.5*(y0+(x[1]-y0)*cos(t)-(x[2]-z0)*sin(t)-x[1])",
"0.5*(z0+(x[1]-y0)*sin(t)+(x[2]-z0)*cos(t)-x[2])"),
y0=0.5, z0=0.5, t=pi/3)
bcl = DirichletBC(V, c, left)
bcr = DirichletBC(V, r, right)
bcs = [bcl, bcr]
v  = TestFunction(V)
u  = Function(V)
B  = Constant((0.0, -0.5, 0.0))
T  = Constant((0.1,  0.0, 0.0))
I = Identity(V.cell().d)
F = I + grad(u)
Ic = tr(F.T*F)
J  = det(F)
E, nu = 10.0, 0.3
mu, lmbda = Constant(E/(2*(1 + nu))),
    Constant(E*nu/((1 + nu)*(1 - 2*nu)))
psi = (mu/2)*(Ic - 3) - mu*ln(J) +
    (lmbda/2)*(ln(J))**2
Pi = psi*dx - dot(B, u)*dx - dot(T, u)*ds
F = derivative(Pi, u, v)

solve(F == 0, u, bcs)
plot(u, interactive=True, mode="displacement")
```

H. Narayanan, *A computational framework for nonlinear elasticity* (2011)

Computational hemodynamics





- Low wall shear stress may trigger aneurysm growth
- Solve the incompressible Navier–Stokes equations on patient-specific geometries

$$\dot{u} + u \cdot \nabla u - \nabla \cdot \sigma(u, p) = f$$
$$\nabla \cdot u = 0$$

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

Computational hemodynamics (contd.)

Python code

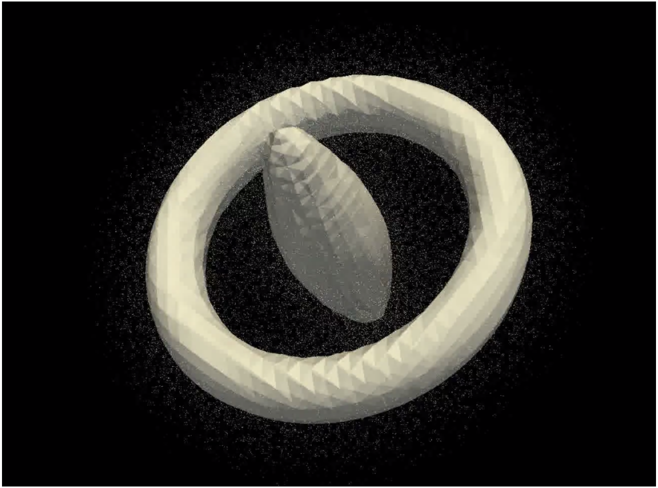
```
# Define Cauchy stress tensor
def sigma(v,w):
    return 2.0*mu*0.5*(grad(v) + grad(v).T)  -
       w*Identity(v.cell().d)

# Define symmetric gradient
def epsilon(v):
    return  0.5*(grad(v) + grad(v).T)

# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx + rho*inner(v,
    grad(u0)*(u0 - w))*dx \
   + inner(epsilon(v), sigma(U, p0))*dx \
   + inner(v, p0*n)*ds - mu*inner(grad(U).T*n,
      v)*ds \
   - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)

# Pressure correction
a2 = inner(grad(q), k*grad(p))*dx
L2 = inner(grad(q), k*grad(p0))*dx - q*div(u1)*dx

# Velocity correction
a3 = inner(v, u)*dx
L3 = inner(v, u1)*dx + inner(v, k*grad(p0 -
    p1))*dx
```

- The Navier–Stokes solver is implemented in Python/FEniCS
- FEniCS allows solvers to be implemented in a minimal amount of code

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

Numerical relativity

$$R_{ab} - \frac{1}{2}Rg_{ab} + g_{ab}\Lambda = 8\pi T_{ab}$$



Ames, Andreasson, Logg, (2015)

Hello World in FEniCS: problem formulation

Poisson's equation

$$
\begin{aligned}
-\Delta u &= f \quad \text{in } \Omega \\
u &= 0 \quad \text{on } \partial\Omega
\end{aligned}
$$

Finite element formulation

Find $u \in V$ such that

$$
\underbrace{\int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f \, v \, \mathrm{d}x}_{L(v)} \quad \forall \, v \in V
$$

Hello World in FEniCS: implementation

Python code

```
from fenics import *

mesh = UnitSquareMesh(32, 32)

V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]")

a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

u = Function(V)
solve(a == L, u, bc)
plot(u)
```

Basic API

- `Mesh`, `Vertex`, `Edge`, `Face`, `Facet`, `Cell`
- `FiniteElement`, `FunctionSpace`
- `TrialFunction`, `TestFunction`, `Function`
- `grad()`, `curl()`, `div()`, ...
- `Matrix`, `Vector`, `KrylovSolver`, `LUSolver`
- `assemble()`, `solve()`, `plot()`

- Python interface generated semi-automatically by SWIG
- C++ and Python interfaces almost identical

Sounds great, but how do I find my way through the jungle?

Three survival advices

Use the right Python tools

Explore the documentation

Ask, report and request

FEniCS PROJECT

About Download Documentation Applications Contributing Citing Support

Documentation for FEniCS 1.3.0

Our documentation includes a book, a collection of documented demo programs, and complete references for the FEniCS application programming interface (API). Note that the FEniCS API is documented separately for each FEniCS component. The most important interfaces are those of the C++/Python problem solving environment *DOLFIN* and the form language *UFL*.

(*This page accesses the FEniCS 1.3.0 documentation. Not the version you are looking for? See all versions.*)

The FEniCS Tutorial

A good starting point for new users is the *FEniCS Tutorial*. The tutorial will help you get quickly up and running with solving differential equations in FEniCS. The tutorial focuses exclusively on the FEniCS Python interface, since this is the simplest approach to exploring FEniCS for beginners.

The FEniCS Book

The FEniCS Book, *Automated Solution of Differential Equations by the Finite Element Method*, is a comprehensive (700 pages) book documenting the mathematical methodology behind the FEniCS Project and the software developed as part of the FEniCS Project. The FEniCS Tutorial is included as the opening chapter of the FEniCS Book.

The FEniCS Manual

The FEniCS Manual is a 200-page excerpt from the FEniCS Book, including the FEniCS Tutorial, an introduction to the finite element method and documentation of DOLFIN and UFL.

Additional Documentation

Mixing software with FEniCS is a tutorial on how to combine FEniCS applications in Python with software written in other languages.

Demos

A simple way to build your first FEniCS application is to copy and modify one of the existing demos:

- Documented DOLFIN demos (Python)
- Documented DOLFIN demos (C++)

The demos are *already installed on your system* or can be found in the `demo` directory of the DOLFIN source tree.

Quick Programmer's References

Some of the classes and functions in DOLFIN are more frequently used than others. To learn more about these, take a look at the

- Basic classes and functions in DOLFIN (Python)
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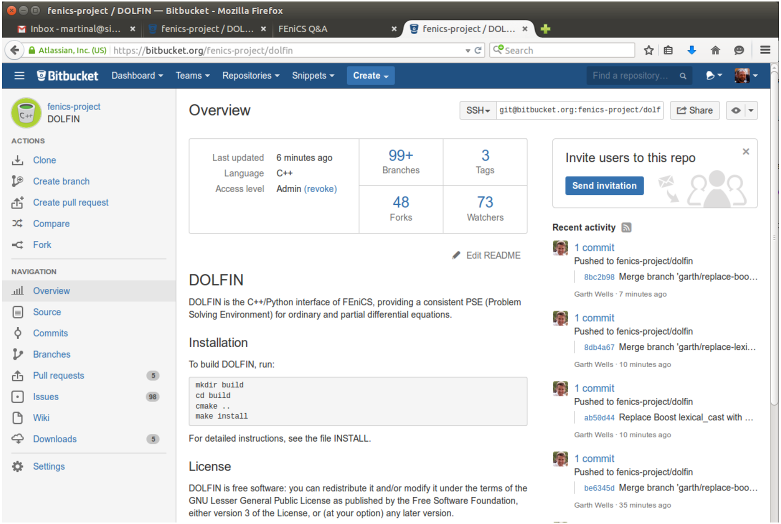
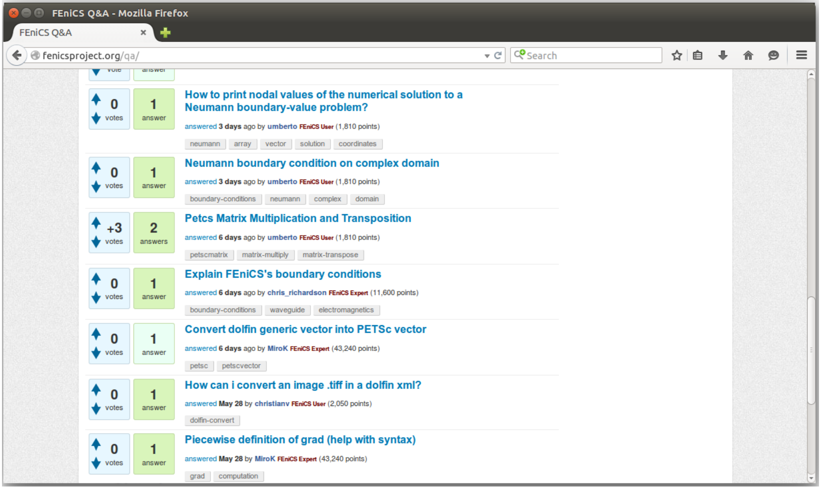
Development community is organized via bitbucket.org



http://bitbucket.org/fenics-project/

Community help is available via QA forum



https://fenicsproject.org/qa

Let's get started and remember:

- **Lectures** can be downloaded from

  `http://fenicsproject.org/pub/course/lectures`

- **Data** for exercises can be downloaded from

  `http://fenicsproject.org/pub/course/data`

  (Or copy from the .../pub/data directory on alarik)

- **Solutions** for exercises can be downloaded from

  `http://fenicsproject.org/pub/course/src`

  (Secret password needed!)