FEniCS Course

Lecture 1: Introduction to FEniCS

Contributors
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What is FEniCS?

FEniCS is an automated programming environment for differential equations

- C++/Python library
- Initiated 2003 in Chicago
- 1000–2000 monthly downloads
- Part of Debian and Ubuntu
- Licensed under the GNU LGPL

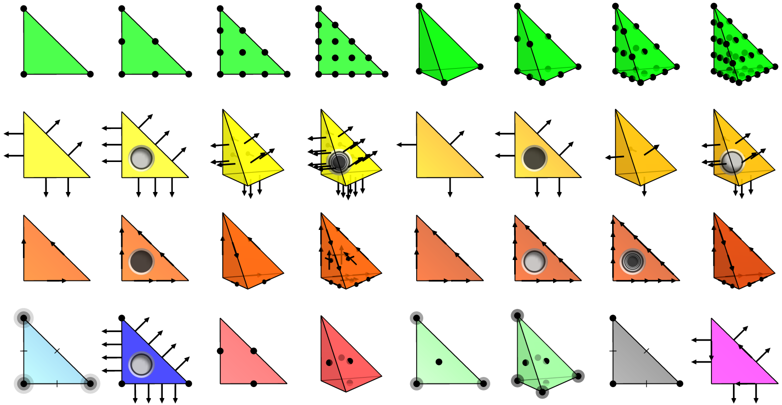
`http://fenicsproject.org/`

Collaborators

Simula Research Laboratory, University of Cambridge, University of Chicago, Texas Tech University, KTH Royal Institute of Technology, Chalmers University of Technology, Imperial College London, University of Oxford, Charles University in Prague, ...

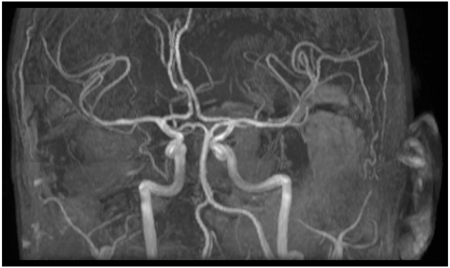
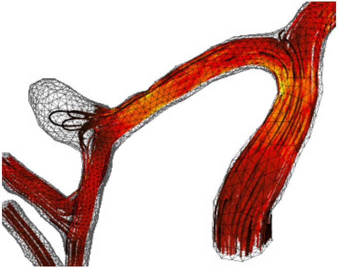
FEniCS is automated FEM

- Automated generation of basis functions
- Automated evaluation of variational forms
- Automated finite element assembly
- Automated adaptive error control



What has FEniCS been used for?

Computational hemodynamics

- Low wall shear stress may trigger aneurysm growth
- Solve the incompressible Navier–Stokes equations on patient-specific geometries

$$\begin{aligned} \dot{u} + u \cdot \nabla u - \nabla \cdot \sigma(u, p) &= f \\ \nabla \cdot u &= 0 \end{aligned}$$

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

Computational hemodynamics (contd.)

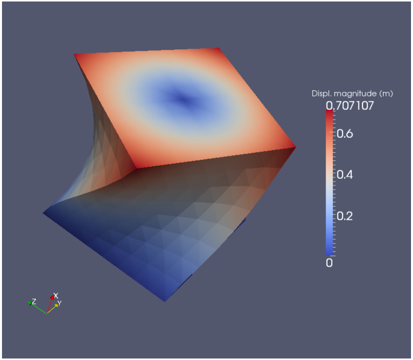
Python code

```
# Define Cauchy stress tensor
def sigma(v,w):
    return 2.0*mu*0.5*(grad(v) + grad(v).T)  -
       w*Identity(v.cell().d)

# Define symmetric gradient
def epsilon(v):
    return  0.5*(grad(v) + grad(v).T)

# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx + rho*inner(v,
    grad(u0)*(u0 - w))*dx \
   + inner(epsilon(v), sigma(U, p0))*dx \
   + inner(v, p0*n)*ds - mu*inner(grad(U).T*n,
      v)*ds \
   - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)

# Pressure correction
a2 = inner(grad(q), k*grad(p))*dx
L2 = inner(grad(q), k*grad(p0))*dx - q*div(u1)*dx

# Velocity correction
a3 = inner(v, u)*dx
L3 = inner(v, u1)*dx + inner(v, k*grad(p0 -
    p1))*dx
```

- The Navier–Stokes solver is implemented in Python/FEniCS
- FEniCS allows solvers to be implemented in a minimal amount of code

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

Hyperelasticity



Python code

```
from fenics import *

mesh = UnitCubeMesh(24, 16, 16)
V = VectorFunctionSpace(mesh, "Lagrange", 1)

left =  CompiledSubDomain("(std::abs(x[0])
    < DOLFIN_EPS) && on_boundary")
right = CompiledSubDomain("(std::abs(x[0] - 1.0)
    < DOLFIN_EPS) && on_boundary")

c = Expression(("0.0", "0.0", "0.0"))
r = Expression(("0.0",
"0.5*(y0+(x[1]-y0)*cos(t)-(x[2]-z0)*sin(t)-x[1])",
"0.5*(z0+(x[1]-y0)*sin(t)+(x[2]-z0)*cos(t)-x[2])"),
y0=0.5, z0=0.5, t=pi/3)
bcl = DirichletBC(V, c, left)
bcr = DirichletBC(V, r, right)
bcs = [bcl, bcr]
v  = TestFunction(V)
u  = Function(V)
B  = Constant((0.0, -0.5, 0.0))
T  = Constant((0.1,  0.0, 0.0))
I = Identity(V.cell().d)
F = I + grad(u)
Ic = tr(F.T*F)
J  = det(F)
E, nu = 10.0, 0.3
mu, lmbda = Constant(E/(2*(1 + nu))),
    Constant(E*nu/((1 + nu)*(1 - 2*nu)))
psi = (mu/2)*(Ic - 3) - mu*ln(J) +
    (lmbda/2)*(ln(J))**2
Pi = psi*dx - dot(B, u)*dx - dot(T, u)*ds
F = derivative(Pi, u, v)

solve(F == 0, u, bcs)
plot(u, interactive=True, mode="displacement")
```

H. Narayanan, *A computational framework for nonlinear elasticity* (2011)

How to use FEniCS?

Hello World in FEniCS: problem formulation

Poisson's equation

$$\begin{aligned} -\Delta u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

Finite element formulation

Find $u \in V$ such that

$$\underbrace{\int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f \, v \, \mathrm{d}x}_{L(v)} \quad \forall\, v \in V$$

Hello World in FEniCS: implementation

Python code

```
from fenics import *

mesh = UnitSquareMesh(32, 32)

V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]")

a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

u = Function(V)
solve(a == L, u, bc)
plot(u)
```

Basic API

- `Mesh`, `Vertex`, `Edge`, `Face`, `Facet`, `Cell`
- `FiniteElement`, `FunctionSpace`
- `TrialFunction`, `TestFunction`, `Function`
- `grad()`, `curl()`, `div()`, ...
- `Matrix`, `Vector`, `KrylovSolver`, `LUSolver`
- `assemble()`, `solve()`, `plot()`

- Python interface generated semi-automatically by SWIG
- C++ and Python interfaces almost identical

Sounds great, but how do I find my way through the jungle?

Three survival advices

Use the right Python tools

Explore the documentation

Ask, report and request

Use the right Python tools!

Python tools

Doc tools

- Standard terminal:
```
> pydoc dolfin
> pydoc dolfin.Mesh
```
- Python console
```
>>> help(dolfin)
>>> help(dolfin.Mesh)
```

Sophisticated Python environments

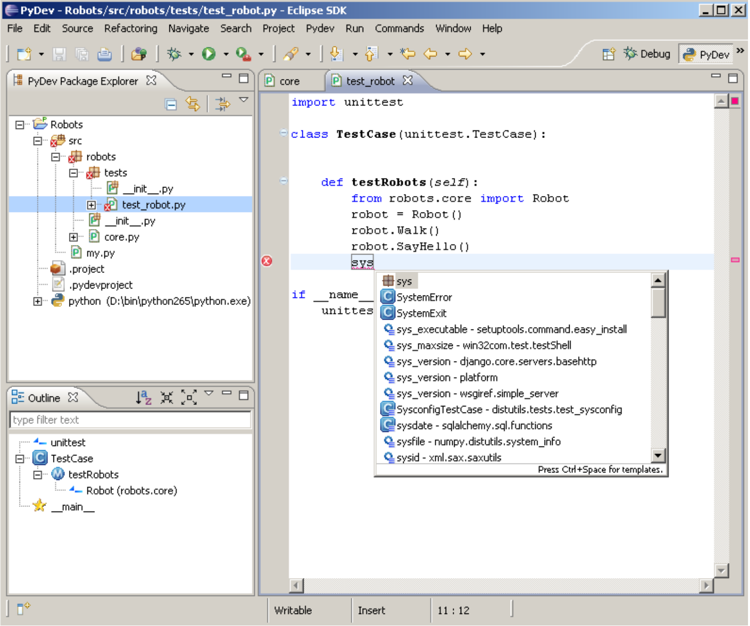
IDLE the official (but rather limited) Python IDE

IPython `http://ipython.org/` provides a Python shell and notebook including syntax highlighting, tab-completion, object inspection, debug assisting, history …

Eclipse plugin `http://pydev.org/` includes syntax highlighting, code completion, unit-testing, refactoring, debugger …

IPython netbook

Eclipse plugin Pydev



Explore the FEniCS documentation!

FEniCS PROJECT

About Download Documentation Applications Contributing Citing Support

Documentation for FEniCS 1.3.0

Our documentation includes a book, a collection of documented demo programs, and complete references for the FEniCS application programming interface (API). Note that the FEniCS API is documented separately for each FEniCS component. The most important interfaces are those of the C++/Python problem solving environment *DOLFIN* and the form language *UFL*.

(*This page accesses the FEniCS 1.3.0 documentation. Not the version you are looking for? See all versions.*)

The FEniCS Tutorial

A good starting point for new users is the *FEniCS Tutorial*. The tutorial will help you get quickly up and running with solving differential equations in FEniCS. The tutorial focuses exclusively on the FEniCS Python interface, since this is the simplest approach to exploring FEniCS for beginners.

The FEniCS Book

The FEniCS Book, Automated Solution of Differential Equations by the Finite Element Method, is a comprehensive (700 pages) book documenting the mathematical methodology behind the FEniCS Project and the software developed as part of the FEniCS Project. The FEniCS Tutorial is included as the opening chapter of the FEniCS Book.

The FEniCS Manual

The FEniCS Manual is a 200-page excerpt from the FEniCS Book, including the FEniCS Tutorial, an introduction to the finite element method and documentation of DOLFIN and UFL.

Additional Documentation

Mixing software with FEniCS is a tutorial on how to combine FEniCS applications in Python with software written in other languages.

Demos

A simple way to build your first FEniCS application is to copy and modify one of the existing demos:

- Documented DOLFIN demos (Python)
- Documented DOLFIN demos (C++)

The demos are *already installed on your system* or can be found in the `demo` directory of the DOLFIN source tree.

Quick Programmer's References

Some of the classes and functions in DOLFIN are more frequently used than others. To learn more about these, take a look at the

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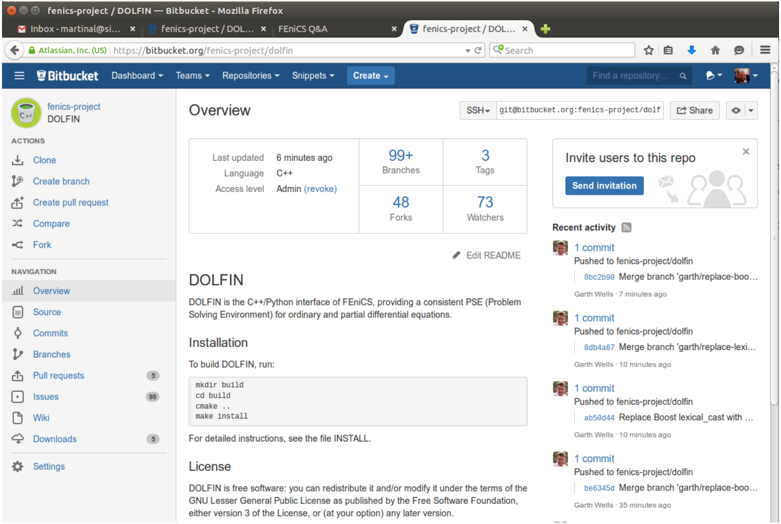
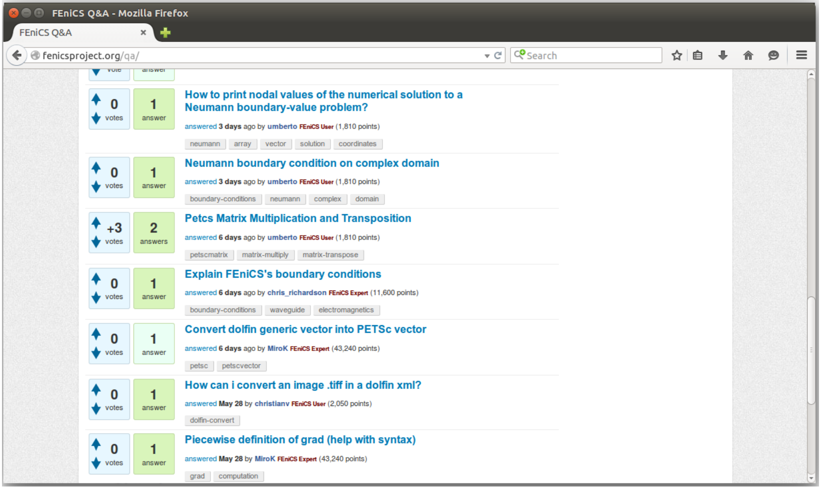
Ask questions, report bugs and request new features!

Development community is organized via bitbucket.org



http://bitbucket.org/fenics-project/

Community help is available via QA forum



https://fenicsproject.org/qa

Installation

- ☞ Use official packages for Debian and Ubuntu
- ☞ Use drag and drop installation on Mac OS X
- ☞ Use VirtualBox + official FEniCS image
- ☞ Build from source (fenics-install.sh)
- ☞ Other options: Docker, Conda packages

http://fenicsproject.org/download/

The FEniCS challenge!

1. Install FEniCS on your laptop!

   http://fenicsproject.org/download/

2. Find and execute `demo_cahn-hilliard.py`, try to visualize the results with Paraview.
3. What are the main packages of the `dolfin` module?
4. Which elements are supported in `dolfin`?
5. Plot at least two finite elements from each row on page 4 and identify those elements you are most curious about!