FEniCS Course

Lecture 1: Introduction to FEniCS

Contributors
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What is FEniCS?

FEniCS is an automated programming environment for differential equations

- C++/Python library
- Initiated 2003 in Chicago
- 1000–2000 monthly downloads
- Part of Debian and Ubuntu
- Licensed under the GNU LGPL

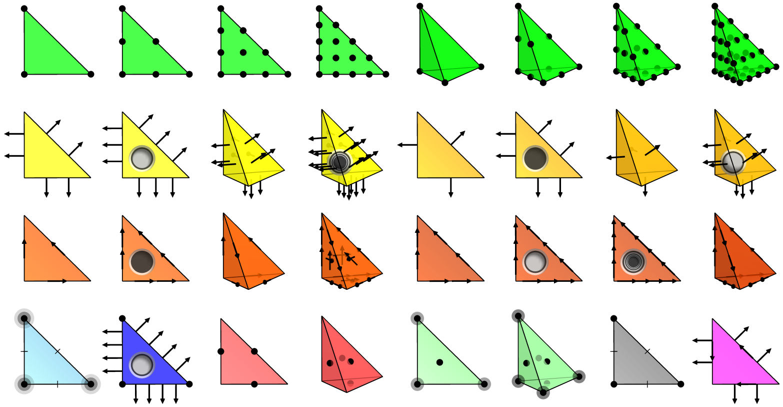
`http://fenicsproject.org/`

Collaborators

Simula Research Laboratory, University of Cambridge, University of Chicago, Texas Tech University, KTH Royal Institute of Technology, Chalmers University of Technology, Imperial College London, University of Oxford, Charles University in Prague, . . .

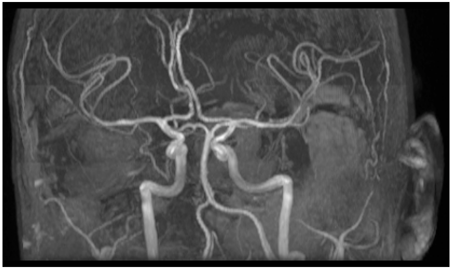
FEniCS is automated FEM

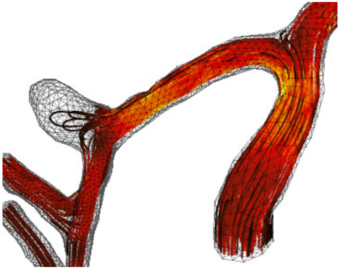
- Automated generation of basis functions
- Automated evaluation of variational forms
- Automated finite element assembly
- Automated adaptive error control



What has FEniCS been used for?

Computational hemodynamics





- Low wall shear stress may trigger aneurysm growth
- Solve the incompressible Navier–Stokes equations on patient-specific geometries

$$\begin{aligned} \dot{u} + u \cdot \nabla u - \nabla \cdot \sigma(u, p) &= f \\ \nabla \cdot u &= 0 \end{aligned}$$

Valen-Sendstad, Mardal, Logg, *Computational hemodynamics* (2011)

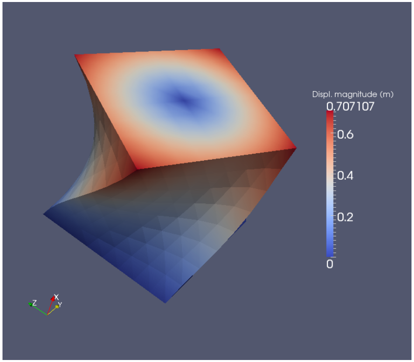
Computational hemodynamics (contd.)

Python code

```
# Define Cauchy stress tensor
def sigma(v,w):
    return 2.0*mu*0.5*(grad(v) + grad(v).T)  -
       w*Identity(v.cell().d)

# Define symmetric gradient
def epsilon(v):
    return  0.5*(grad(v) + grad(v).T)

# Tentative velocity step (sigma formulation)
U = 0.5*(u0 + u)
F1 = rho*(1/k)*inner(v, u - u0)*dx + rho*inner(v,
    grad(u0)*(u0 - w))*dx \
   + inner(epsilon(v), sigma(U, p0))*dx \
   + inner(v, p0*n)*ds - mu*inner(grad(U).T*n,
       v)*ds \
   - inner(v, f)*dx
a1 = lhs(F1)
L1 = rhs(F1)

# Pressure correction
a2 = inner(grad(q), k*grad(p))*dx
L2 = inner(grad(q), k*grad(p0))*dx - q*div(u1)*dx

# Velocity correction
a3 = inner(v, u)*dx
L3 = inner(v, u1)*dx + inner(v, k*grad(p0 -
    p1))*dx
```

- The Navier–Stokes solver is implemented in Python/FEniCS
- FEniCS allows solvers to be implemented in a minimal amount of code

Hyperelasticity



Python code

```
from fenics import *

mesh = UnitCubeMesh(24, 16, 16)
V = VectorFunctionSpace(mesh, "Lagrange", 1)

left =  CompiledSubDomain("(std::abs(x[0])
    < DOLFIN_EPS) && on_boundary")
right = CompiledSubDomain("(std::abs(x[0] - 1.0)
    < DOLFIN_EPS) && on_boundary")

c = Expression(("0.0", "0.0", "0.0"))
r = Expression(("0.0",
"0.5*(y0+(x[1]-y0)*cos(t)-(x[2]-z0)*sin(t)-x[1])",
"0.5*(z0+(x[1]-y0)*sin(t)+(x[2]-z0)*cos(t)-x[2])"),
y0=0.5, z0=0.5, t=pi/3)
bcl = DirichletBC(V, c, left)
bcr = DirichletBC(V, r, right)
bcs = [bcl, bcr]
v   = TestFunction(V)
u   = Function(V)
B   = Constant((0.0, -0.5, 0.0))
T   = Constant((0.1,  0.0, 0.0))
I = Identity(V.cell().d)
F = I + grad(u)
Ic = tr(F.T*F)
J  = det(F)
E, nu = 10.0, 0.3
mu, lmbda = Constant(E/(2*(1 + nu))),
    Constant(E*nu/((1 + nu)*(1 - 2*nu)))
psi = (mu/2)*(Ic - 3) - mu*ln(J) +
    (lmbda/2)*(ln(J))**2
Pi = psi*dx - dot(B, u)*dx - dot(T, u)*ds
F = derivative(Pi, u, v)

solve(F == 0, u, bcs)
plot(u, interactive=True, mode="displacement")
```

H. Narayanan, *A computational framework for nonlinear elasticity* (2011)

How to use FEniCS?

Hello World in FEniCS: problem formulation

Poisson's equation

$$\begin{aligned} -\Delta u &= f \quad \text{in } \Omega \\ u &= 0 \quad \text{on } \partial\Omega \end{aligned}$$

Finite element formulation

Find $u \in V$ such that

$$\underbrace{\int_\Omega \nabla u \cdot \nabla v \, \mathrm{d}x}_{a(u,v)} = \underbrace{\int_\Omega f \, v \, \mathrm{d}x}_{L(v)} \quad \forall \, v \in V$$

Hello World in FEniCS: implementation

Python code

```
from fenics import *

mesh = UnitSquareMesh(32, 32)

V = FunctionSpace(mesh, "Lagrange", 1)
u = TrialFunction(V)
v = TestFunction(V)
f = Expression("x[0]*x[1]")

a = dot(grad(u), grad(v))*dx
L = f*v*dx

bc = DirichletBC(V, 0.0, DomainBoundary())

u = Function(V)
solve(a == L, u, bc)
plot(u)
```

Basic API

- `Mesh`, `Vertex`, `Edge`, `Face`, `Facet`, `Cell`
- `FiniteElement`, `FunctionSpace`
- `TrialFunction`, `TestFunction`, `Function`
- `grad()`, `curl()`, `div()`, ...
- `Matrix`, `Vector`, `KrylovSolver`, `LUSolver`
- `assemble()`, `solve()`, `plot()`

- Python interface generated semi-automatically by SWIG
- C++ and Python interfaces almost identical

Installation

Official packages for Debian and Ubuntu

Drag and drop installation on Mac OS X

(Binary installer for Windows with old version 1.0)

Automated installation from source

http://fenicsproject.org/download/

The FEniCS challenge!

Install FEniCS on your laptop!

`http://fenicsproject.org/download/`

Does it work?

Python code

```
from fenics import *

mesh = UnitCubeMesh(16, 16, 16)
plot(mesh)
interactive()
```